C.U.

PHYSICS HONOURS QUESTION PAPERS (PART–I & II)—2017

PART – I

FIRST PAPER – 2017

Full Marks – 100

Answer *Question No. 1* and *any four* questions from each Unit

1. Answer *any ten* of the following : 2×10

(a) Find the values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n+1}$ is convergent.

(b) Prove that $\int_v (\phi\nabla^2\psi + \nabla\phi.\nabla\psi)dV = \oint(\phi V\psi)d\bar{S}$ where V is the volume enclosed by the closed surface S.

(c) Prove that the product of two Hermitian matrics A and B is Hermitian only if A and B commute.

(d) Using the generating function $\phi(x,h) = (1-2xh+h^2)^{-1/2}$

$= \sum_{l=0}^{\infty} h^l P_l(x), (|h|<1)$ find out the Legendre Polynomial $P_2(x)$.

(e) Find the Fourier transform of $\delta(x-a)+\delta(x+a)$ where a is a constant.

(f) Find whether $d\phi$ is an exact differential where

$d\phi = (x^3 - 4xy)\,dx + (y^2 - 2x)dy$.

(g) Distance between two points in a medium is $2m$. The optical path corresponding to this distance is $2.5m$. Find out velocity of light in the medium.

(h) Distinguish between amplitude resonance and velocity resonance.

(i) Draw a sketch of the drain characteristics of a MOSFET and identify the different regions.

(j) Verify the Boolean identity

$$AC + ABC = AC$$

(k) Why FET's can be used at higher frequencies than BJT's?

(l) A dice is thrown 8 times. What is the probability that 4 will show (i) exactly twice (ii) at least 6 times ?

Unit - I

2. (a) (i) Deduce $\vec{\nabla}\times(\phi\vec{A}) = (\vec{\nabla}\phi)\times\vec{A} + \phi(\vec{\nabla}\times\vec{A})$.

(ii) State Stokes' theorem for a vector $\vec{V}$. Let $\vec{V} = \phi\vec{c}$ where $\vec{c}$ is a constant vector. Use the result of (i) to deduce that $\int \vec{\nabla}\phi \times d\vec{s} = \oint \phi d\vec{r}$. 2+1+3

(b) Given the vector $\vec{A} = (x^2 - y)\hat{i} + 2x\hat{j} + 2\hat{k}$, evalute $\oint \vec{A}.d\vec{r}$ around the boundary of the circle $x^2 + y^2 = 1$. 4

3.(a) Show that any square matrix can be written as the sum of a symmetric and antisymmetric matrix. 2

(b) Find the eigenvalues and the normalised eigenvector of the matrix $M = \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$. 2+3

(c) A normal matrix N is defined by the relation $NN^+ = N^+ N$. If N is written as $(A + iB)$, where A and B are Hermitian, show that A and B commute. 3

4. (a) For free path of length x during which a molecule of an ideal gas does not suffer any collision with another molecule, the probability distribution function is given by $P = \frac{1}{\lambda}e^{-x/\lambda}$, $0 < x < \infty$. Show that the mean free path is λ. 3

(b) Expand In $(1 + x)$ in a Taylor series about the point $x = 0$. Hence write the n^{th} term of the series. 3

(c) Evalute $\iint xy\, dxdy$ over the area between $y = x^2$ and $y = x$. 4

5. (a) Apply Frobenius method to solve the equation

$$\frac{d^2y}{dx^2} + y = 0, \text{ setting } y(x) = \sum_{\lambda=0}^{\infty} a_\lambda x^{k+\lambda} \text{ with } a_0 \neq 0.$$

(i) Verify that the indicial equation is $k(k-1) = 0$. (ii) For $k = 1$, show that a_1 is necessarily zero. (iii) Find the recurrence relation. (iv) Hence show that for $k = 1$, the solution is $y = a_0 \sin x$. 2+1+2+2

(b) A particle moves in a plane such that

$$\frac{dx}{dt} = -50y \text{ and } \frac{dx}{dt} = 18x.$$

Given that it passes through the point (2, 0), find the equation of the path. 3

6. (a) Solve $\frac{\partial U}{\partial x} = 4\frac{\partial U}{\partial y}$ by the method of separation of variables, given that $U(0, y) = 8e^{-3y}$.

(b) Given

$$e^{-t^2+2tx} = \sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!}$$

where H_n is the Hermite Polynomial, show that $H_n(x) = (-1)^n H_n(-x)$.

(c) A plucked string of length L is excited at $x = L/3$ and touched at $x = L/4$. Calculate the harmonics present. 4

7. (a) Expand the function

$$f(x) = x^2 \text{ for } -\pi < x < \pi$$
$$f(x + 2\pi) = f(x)$$

in a Fourier series, Hence prove, $\frac{\pi^2}{6} = \sum_{n=1}^{\infty}\frac{1}{n^2}$. 4+2

(b) Show that the Fourier transform of $f(x) = e^{-|x|}$ is $F(K) = \sqrt{\frac{2}{\pi}}\frac{1}{1+k^2}$. 4

Unit – II

8. (a) Deduce the Lagrange-Helmholtz relation connecting lateral and angular magnification of an optical system. 4

(b) Using the paraxial approximation, construct the system matrix for a thin lens made with a material of RI μ and radii of curvature R_1 and R_2 respectively. 4

(c) Show that the planar surface of a plano-convex lens does not contribute to the system matrix. 2

9. (a) A mass of 1 kg is acted on by a restoring force with force constant 4N/m and a resisting force with damping coefficient 2N-s/m. Write down the equation of motion in one dimension. Find :

(i) whether the motion is peridoic or oscillatory. (ii) the value of the resisting force which will make the motion critically damped. 1+2+2

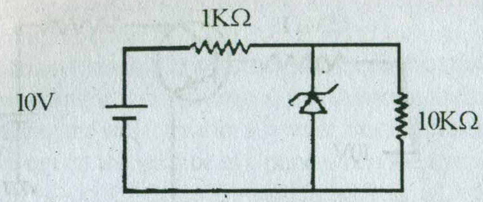
(b) Define group velocity and phase velocity. 2

(c) The dispersion relation for transverse waves propagating in a medium is given by $\omega^2 = \omega_p^2 + k^2c^2$ where the symbols have their usual meanings. Show that $v_g v_p = c^2$. 3

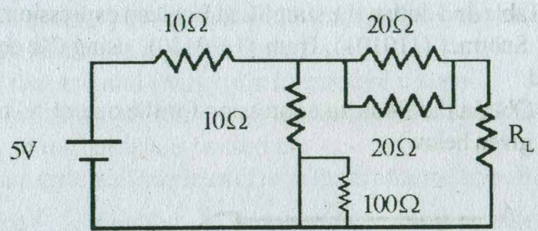
10. (a) A particle is subjected to two SHMs at right angles to each other having the same frequency. Show that the resultant locus of the particle is an ellipse. Hence find the locus when the two motions are in phase and in the opposite phase. 3+1+1

(b) A number of SHMs, all in the same straight line and having equal amplitude and frequency but an equal phase difference θ between the consecutive SHMs are superimposed. Calculate the amplitude and phase of the resultant motion using complex form of representation. 5

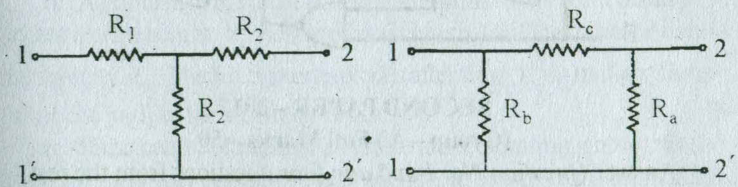
11. (a) In the circuit below, what are the currents that flow through 1KΩ, 10 KΩ resistors and the Zener diode ? Assume that the Zener has a breakdown voltage 6V, what will be the value of current if the 10 KΩ resistor is replaced by 500 Ω resistor? What is the voltage across the Zener now? Consider the Zener to be an ideal device. 2+2+1



(b) Calculate the voltage drop and power dissipated R_L in the circuit below, using Thevnin's theorem. Find the value of R_L for which power dissipated across R_L is maximum. 4+1



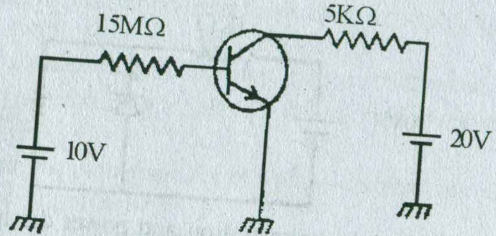
12.(a) Consider the T and π networks.



- Show that the networks will be equivalent when

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, R_2 = \frac{R_c R_a}{R_a + R_b + R_c}, R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \quad 5$$

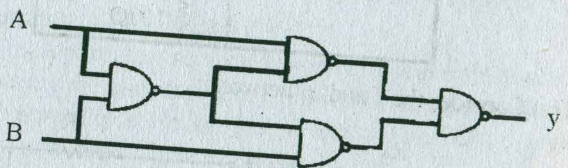
(b) For a transistor in CE configuration given below, it is found that for a fixed base current of 30 μA, I_C changes from 3.5mA to 3.7mA when V_{CE} changes from 7.5V to 12.5V. Calculate its output resistance and β at V_{CE} = 12.5V. What will be the value of α ? 5



13.(a) A function of 3 Boolean variables have high output either when all 3 inputs are high or any one input is high. Write down the Truth Table and derive the simplified Boolean expression.

(b) Subtract $(110101)_2$ from $(1110110)_2$ using 2's complement method. 2

(c) Obtain the Boolean expression for the output 'y' of the logic circuit given below. 4



SECOND PAPER – 2017

(Group – A) Full Marks – 50

Answer *Question No. 1* and *any four* questions from the rest

1. Answer *any five* of the following : 2×5

(a) A particle is under the influence of a force $\bar{F}$ and has an instantaneous velocity $\bar{V}$. Show that $\frac{dT}{dt} = \bar{F}.\bar{V}$, where T is the kinetic energy.

(b) The angular momentum of a particle is $\bar{l}\,(t) = f(t)\,\hat{n}$ where f(t) is a function of time and $\hat{n}$ is some constant unit vector. Show that the motion of this particle is restricted to a plane.

(c) "Angular momentum and angular velocity of a rigid body are not always parallel" – Justify the statement.

(d) What is the significance of the total area under the Maxwellian velocity distribution curve ?

(e) Write down Planck's law for the energy distribution of black body radiation. Plot the distributions at two different temperatures.

(f) How much time will it take for a layer of ice of thickness 20cm to increase by 10 cm on the surface of a pond when the temperature of the surrounding is – 15°C? Given K = 0.005 C.G.S. unit, L = 80 cal/gm, $\rho = 0.9$ gm/cm^3.

2. A particle of mass m moves along a trajectory given by $x = x_0 \cos\omega t$, $y = y_0 \sin\omega t$.

(a) Find the x and y component of the force. What is the condition under which the force is a central force ? 2+2

(b) Find the potential energy as a function of x and y. 2

(c) Determine the kinetic energy of the particle. Show that the total energy of the particle is conserved. 2+2

3. (a) For a system of particles of total mass M acted upon by total external force $\bar{F}_{ext}$, show that $\bar{F}_{ext} = M\bar{a}$, where $\bar{a}$ is the acceleration of the centre of mass. 3

(b) A uniform cylindrical firecraker of mass 'm' is ignited and projected from a height 'h' (centre of mass height) with an intial horizontal velocity $\bar{u}_0$. The firecraker explodes after time 't_e' in mid-air. Determine the path of the centre of mass. 2

(c) If the centre of mass of a system is accelerating, show that the total torque about centre of mass due to the pseudoforce arising out of the acceleration of the centre of mass is zero.

Hence, show that for a system of particles

$$\bar{T} = \frac{d\bar{L}'}{dt}$$

with $\bar{T} = \sum_i \bar{r}_i' \times \bar{F}_i$ and $\bar{L}' = \sum_i \bar{r}_i' \times \bar{p}_i'$. Here, $\bar{r}_i'$ and $\bar{p}_i'$ are the position and momentum of the i-th particle with respect to the centre of mass but $\bar{F}_i$ is the true total force on the i-th particle as seen from an inertial frame. Under what condition can you replace the torque in the above equation by the external torque on the system ? 2+2+1

4. (a) Find the moment of inertia of a rigid body about an arbitrary axis having direction cosines α_1, α_2 and α_3 with respect to X, Y and Z axes, respectively and passing through the origin. What is ellipsoid of inertia ? 4+2

(b) A homogeneous square lamina is placed on the x-y plane with its centre at the origin and edges parallel to the x and y axes. Now the coordinate system is rotated by some angle ϕ about the z-axis. Show that the resulting coordinate system is a principal axes system irrespective of the angle ϕ. 4

5. (a) The relative motion of two molecules can be described as the motion of a single particle reduced mass $m_1 = \frac{m}{2}$. Using this idea we get the distribution of the relative velocities of the molecules

$$dn_{C_r} = n\left(\frac{m}{4\pi KT}\right)^{3/2} \exp\left(-\frac{mC_r^2}{4KT}\right) 4\pi C_r^2 dC_r$$

Where C_r is the relative velocity of two molecules. Show that the average relative velocity of the molecules is $\overline{C}_r = \sqrt{2}\,\overline{c}$, where $\overline{c}$ is the average velocity of the molecules. 4

(b) A shower of 5000 molecules of a gas, each moving with the same velocity initially, traverses a gas. Find the number of molecules that would travel undeflected even after travelling a distance equal to the mean free path. Also, find the number of molecules having free path lying within λ and 2λ, where λ is the mean free path. 2+1

(c) Prove that coefficient of viscosity of a gas is independent of pressure over a wide range. Why is this no longer valid at extremely low pressure ? 2+1

6. (a) Colloidal particles are suspended in liquid. Using Einstein's theory, find the temperature dependence of the mean square displacement per unit time. 6

(b) A particle under Brownian motion at 27°C has an r.m.s speed

1m/sec.Find the mass of the particle. Boltzmann Constant $K = 1.38 \times 10^{-23}$ J/K. 4

7. (a) Express van der Walls equation of state of a real gas in virial form. Determine the Boyle temperature from it. 2+1

(b) A long wire resistivity 2×10^4 ohm – cm and 1 mn in diameter carries a current of 5 ampere. If it is covered uniformly with a cylindrical layer of insulating material having coefficient of thermal conductivity of 6×10^{-4} c.g.s unit and outer diameter of 1 cm, what is the temperature difference between the inner and outer surface of the insulating layer at steady state ? 4

(c) What is solar constant ? Each square meter of sun's surface radiates energy at the rate of 6.3×10^7 Joule/m²/s, and Stefan's Constant is 5.67×10^{-8} W/m²/K⁴. Find the temperature of the surface of the sun. 1+2

THIRD PAPER – 2017

Full Marks – 100

Question No. 1 is compulsory. Answer ***any eight*** questions, taking ***four*** from each ***unit***

1. Answer ***any ten*** of the following questions : 2×10

(a) An amplifier with mid gain $|A| = 500$ has negative feedback $|\beta| = 0.01$. If the upper cut-off frequency without feedback were at 50 kHz, then calculate its value with feedback.

(b) A magnetic field $\overline{B} = \hat{i} + 2\hat{j} - 4\hat{k}$ T and an unknown electric field $\overline{E}$ exists in a region. If an electron moving within that region with velocity $\overline{v} = 2(3\hat{i} + \hat{j} - 2\hat{k})$ m/s experiences no force, calculate $\overline{E}$.

(c) How a J-K Flip-flop can be converted to D Flip-flop ?

(d) A current distribution gives rise to the magnetic vector potential $\overline{A}(x, y, z) = x^2 y\hat{i} + y^2 x\hat{j} - xyz\hat{k}$ T-m. Find the corresponding magnetic field $\overline{B}$ at (–1,2,5).

(e) Check whether the LCR series combination with L = 0.2H, C= 2μF , R = 100 Ω connected to a DC source will be oscillatory in nature ? Plot the current through the circuit as a function of time when the source is suddenly connected.

(f) An AC circuit connected to a 220V. 50Hz supply contains a 20H coil of resistance 100 Ω connected in series with 1μF capacitor. Calculate the power factor of circuit.

(g) Determine the electric field due to the potential $\Phi(r) = (A/r)\,e^{-\lambda r}$.

(h) Consider an infinite straight chain of alternating charges +q and –q. The distance between the alternating charges is 'a'. Find the interaction energy of each charges with all the others.

(i) Find the dimension of $\sqrt{\varepsilon_o / \mu_o}$.

(j) If a zone plate is constructed in such a way that the radii of the zones are $\sqrt{nk}$ and even zones (with even value of n) are blackened, show the points at the distance $\frac{k^2}{m\lambda}$ will be maximum when $m = 1$ and minimum when $m = 2$.

(k) A reflected light emerging after incidence at Brewster's angle is plane polarized. A $\frac{\lambda}{4}$ plate placed in reflected light can transform it into a circularly polarized light. Mention the orientation of such $\frac{\lambda}{4}$ plate.

(l) What is Brewster's law ?

Unit – 5

2.(a) Find the output voltage for the given circuit configuration. 4

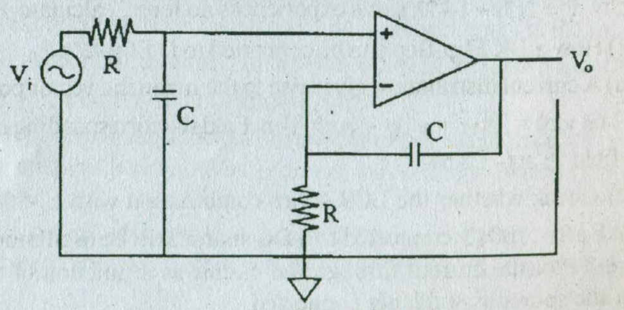


Figure 3

is represented by $v = v_o \cos(\omega_c t + m_f \sin\omega_m t)$, where the symbols have their usual meanings. Show that if the modulation index $m_f << 1$, then the band width of the modulated wave is approximately $2\omega_m$. 1+3

(c) The highest audible frequency is 18 kHz. If the modulation index of an FM wave is 4.5, find the minimum bandwidth required for the detection of the FM wave. 2

3.(a) A symmetrical astable multivibrator using bipolar junction transistor (β = 90) is designed for the frequency 20kHz. Calculate the value of R_C, R_B and the capacitor C. It is given that supply voltage 12 volt, the collector saturation current 6 mA and T = 1.38 R_BC. Consider that the transistors are ideal and just saturate in ON state. 4

(b) Show that for a signle tuned amplifier total output impedance, Z becomes approximately.

$$Z \approx \frac{R_T}{1 + j2Q\delta}$$

when the operating frequency is very near to the tuned frequency ω_0. Here $\delta = \frac{\omega - \omega_0}{\omega_0}$ and $Q = \frac{R_T}{\omega_0 L} = \omega_0 R_T C$. The L and C are the inductor and capacitor of the tank circuit and R_T is the total output resistance. 4

(c) What is the utility of using Radio Frequency Choke in the collector for Hartley and Colpitts oscillators ? 2

4.(a) Draw a clocked S-R flip flop circuit using two-input NAND gates and explain its operation with proper state table. 2+2

(b) Explain how a clocked S-R flip flop is converted into a delay flip flop. 2

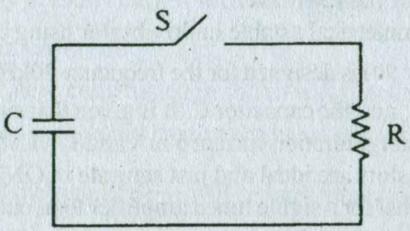
(c) (i) Discuss the principle of operation of a 4-bit digital comparator.

(ii) Distinguish between combinational and sequential logic circuits. 2+2

5.(a) Write down the Biot-Savart law for the general case of a volume current and verify that $\nabla.\bar{B} = 0$. 3

(b) Find the field at the centre of a regular hexagon carrying a steady current I. Let R be the distance from the centre of the hexagon to any side. 4

(c) For the C–R circuit shown below if V_o is the voltage across the capacitor initially, then show that the electrical energy stored in C is entirely dissipated in R after the switch S is closed.



6.(a) A long and uniform cylinder of radius R, carries a magnetization parallel to the axis $\overline{M} = kr\hat{z}$. Here k a constant and r is the distance from the axis. There is no free current anywhere. Using Ampere's law, find $\overline{H}$ and $\overline{B}$ inside and outside the cylinder. 4

(b) In the interior of a permanent magnet $\overline{H}$ and $\overline{B}$ have opposite directions. Which parts of the hysteresis loop correspond to such a magnetic state ? 2

(c) How does a magnetic circuit differ from electric circuit? A magnetic circuit consists of a circular iron-core of permeability μ, a mean radius R and an air gap of width d. Cross section of the core is A and the number of turns of the coil is N carrying current I. Derive an expression for the intensity of magnetic field in the air gap. 4

7.(a) The voltage magnification in an LCR circuit is defined as

$$\frac{\textit{p.d across the inductance (or capacitance)}}{\textit{applied emf}}$$

$C = 0.5\,\mu F$ with $R = 10\,\Omega$, calculate the voltage magnification in the circuit at reasonant frequency. 2

(b) Calculate the impedance for the following circuit and find the frequency for which the current in this circuit will be in phase with the input voltage.

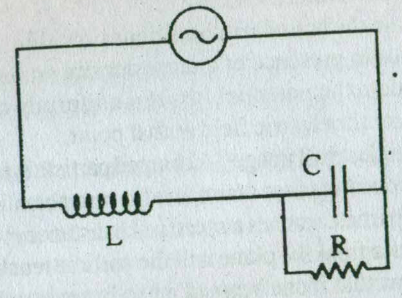


Figure 5

Also find the value of frequency for which the impedance will become inductive only. 2+1+2

(c) Two long parallel wires are separated in air by a distance d. Current passes through them in opposite sense. Radius of each wire is a. Calculate the self-inductance per unit length when d>>a. 3

Unit – 6

8.(a) The electric field in the x-y plane is given by : $\overline{E} = 2ax\hat{i} + by\hat{j}$ V/m. What is the charge density responsible for this field ? 3

(b) A conical surface (an empty ice-cream cone) carries a uniform surface charge density σ. The height of the cone is a, and R is the radius at the top. Find the potential difference between the vertex and the centre of the top. 4

(c) The electric field in a region of space is given by $\overline{E} = 8x\hat{i} - 4y\hat{j} - 4z\hat{k}$. Find the equation of the lines of force in the plane z = 0. 3

9.(a) If a charge distribution is spherically symmetric, show that all but the monopole term of the multipole expansion corresponding to this distribution vanish. Show that the dipole moment of a system of charges is independent of the choice of the origin if the net monopole moment is zero. 3

(b) A circular disc of radius a on the xy plane has a surface charge density, $\sigma = \sigma_0 r \cos\theta / a$. Calculate the electric dipole moment of this charge distribution. 4

(c) Determine the force acting on an electric dipole placed within an inhomogeneous electric field. 3

10.(a) State the boundary conditions prevaling at the interface of two dielectrics in presence of charge density on the interface. 3

(b) Calculate the potential inside a uniformly charged spherical shell. Calculate the electric field at that point. 2+1

(c) What is electrical image ? A charged particle is at a distance d from an infinite conducting plane maintained at zero potential. When released from rest, the particle reaches a speed u at a distance $d/2$ from the plane. At what distance from the plane will the particle reach the speed $2u$? 4

11.(a) Show that if one beam of a two beam interference setup has an irradiance of N times that of the other beam, the fringe visibility is given by

$$V = \frac{2\sqrt{N}}{N+1}$$ 2

(b) In the Newton's ring arrangement, write down the conditions of maxima and minima for both the reflected and transmitted light, Compare these two fringes. 4

(c) Newton's rings are formed between a spherical lens surface and an optical flat. If the tenth bright ring of green light (546.1 nm) is 7.89 mm in diameter, what is the radius of curvature of the lens surface ? 2

(d) Show that fringe width of the interference pattern produced by Fresnel's biprism is expressed as $\beta = \left(1+\frac{b}{a}\right)\times\frac{\lambda}{2\alpha(n-1)}$

where a = distance between slit and biprism, b = distance between biprism and screen, α = angle of biprism and n = refractive index of the material of the prism. 2

12.(a) Distinguish between single slit and double slit diffraction

(b) State and explain Rayleigh criterion of resolution. Find the separation of two points on the moon that can be resolved by a 500 cm telescope. The distance of the moon is 3.8×10^5 km. The eye is most sensitive to light of wavelength 5500 Å. 3+2

(c) Red orange lines of Neon are identified with the wavelength 6217Å. and 6266Å If a grating with 10,000 lines per inch is allowed to received the light. What will be separation of these lines in the third order of spectrum at the focal plane of a convex lens of the focal length 25 cm. 3

13. (a) Explain the phenomenon of double refraction in a uniaxial crystal by applying Huygen's theory. 2

(b) Imagine two crossed linear polarizers with transmission axes vertical and horizontal. Now insent a third linear polarizer between them with transmission axis at 45° to the vertical. Determine the emerging irradiance in terms of incident irradiance.

(c) The rotation in the plane of polarization at wavelength 5893 Å, in a certain substance is 10°/cm. Calculate the difference between the refractive indices for right and left circular polarized light in the medium. 2

(d) A plane polarized light of wavelength 550 nm changes to circularly polarized light on passing through a quartz crystal cut parallel to the optic axis. Calculate the minimum thickness to produce such effect. Given $(n_e - n_0) = 0.005$ where n_e and n_0 are the refreactive indices for E-ray and O-ray respectively. 3

FOURTH PAPER– 2017

GROUP-A

Full Marks – 50

Answer **Question No. 1** and **any four** from the rest

Symbols have their usual meaning everywhere

Given h = 6.626×10^{-34} J -sec

m= 9.1×10^{-31} kg

c = 3×10^{8} m/sec

1. Answer ***any five*** of the following :

(a) The human eye can detect 1.0×10^{-18} Joules of electromagnetic ... How many 600nm photons does this represent ? 2

(b) If the uncertainty in the position of a particle be determined up to an accuracy of 10^{-8} m, what is the corresponding uncertainty in momentum ? 2

(c) Starting from the basic commutation relation $[x, p_x] = i\hbar$, one can show that $[x, p_x^n] = i\hbar n p_x^{n-1}$. Using this result or in other way, prove that $[x, \sin p_x] = i\hbar \cos p_x$. 2

(d) If the state functions p, V and T are related by an equation of state f (p, V, T) = 0, then show that

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = -1$$ 2

(e) A 100 Ω resistor is held at constant temperature of 300 K. A current of 10 ampere is passed through the resistor for 300 sec. What is the change in enfropy of the resistor and the universe ? 2

(f) Why the slope of the fusion curve in the P – T phase diagram of water is negative ? 2

2.(a) Using the concept of discrete energy and Boltzman probability distribution, find the average energy per mode of oscillation of an assembly of harmonic oscillators. Hence, derive the spectral distribution function assuming the number of modes per unit volume per unit wavelength to be $\left(8\pi/\lambda^4\right)$. What is the name of the spectra distribution obtained above ? 3+1

(b) Repeat the above treatment for the case of continuous variation in energy. Mention the name of the spectral distribution obtained. 3+1

(c) A layer of Sodium 1 atom thick and 1 m^2 in area contain 10^{19} atoms. If the incident light delivers a power of 10^{-25} watts to each atom, then what would be the time required for an atom to accumulate sufficient energy to emit a photo electron classically (Given work function for sodium = 2.3eV). What inference can be drawn regarding the Einstein's Photoelectric effect from the above problem ? 2

3.(a) An electron and a proton have the same kinetic energy. Compare the wavelengths and the phase and group velocities of their de Broglie waves. 3

(b) Write down the time independent Schrö ndinger equation for a partcle moving in 1 – D. If there is no force acting on it, find out the solution. If the particle is constrained to move such that $0<x<L$, what are the boundary conditions of the wave function ? 3

(c) Using the commutation relation for the components of the momentum and position operators, show that the components of the angular momentum operator *L* satisfy

$$[L_x L_y] = i\hbar L_z.$$ 4

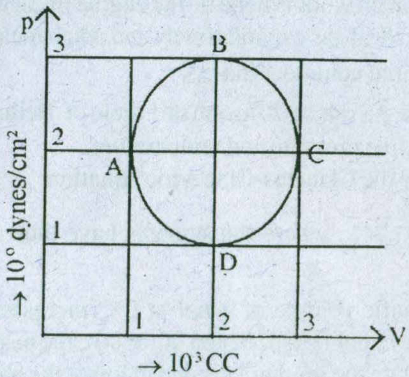
4.(a) Show that for an operator $\hat{A}$ with no explicit time independence :

$$\frac{d\hat{A}}{dt} = \frac{i}{\hbar}\left\langle\left[\hat{H}, \hat{A}\right]\right\rangle.$$ Hence, prove that $\frac{d}{dt}\langle p_x\rangle = -\left\langle\frac{dV(x)}{dx}\right\rangle$ for a particle moving in x direction with momentum p_x under the potential V(x). 3+3

(b) Prove Ehrenfest theorem in one dimension

$$\frac{d}{dt}\langle x\rangle = \frac{\langle p_x\rangle}{m}.$$ 4

5.(a) Suppose that 1 mole of an ideal gas is subjected to a cyclic quasi-static process, which appears as a circle in pV diagram.



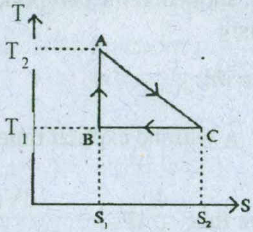
Show that

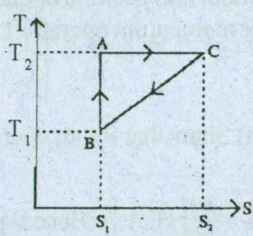
(i) the net work done by the gas in one cycle is about 314 Joules.

(ii) the internal energy difference of the gas between state C and state A is 600 Joules.

(iii) the heat absorbed by the gas in going from A to C via state B is about 1157 Joules. 2+2+2

(b) Compare the efficiencies of the cycles ABCA shown in figures below : 4





6.(a) Is the isothermal expansion of an ideal gas violates the second law of thermodynamics ? 2

(b) A Carnot engine is operated between the ice-point and steam-point.

(i) If engine receives 746 cal from the hot reservoir in each cycle, how many calories does it reject to the cold reservoir ? (ii) If the engine is operated as a refrigerator and receives the same heat from the cold reservoir, how many calories does it deliver to the hot reservoir? (iii) How much work is done by the engine in each case ? 1+1+1

(c) 1 mole of ideal gas expands freely and adiabatically in vacuum to twice of its initial volume. Find ΔS. 3

(d) Calculate ΔS due to diffusion of 1 mole of Helium and 1 mole of Nitrogen at same pressure and temperature. 2

7.(a) Deduce the Clausius-Clapeyron equation

$\frac{dp}{dT} = \frac{L}{T(V_2 - V_1)}$, where the symbols have their usual significance. 4

(b) The specific volume of water at 0°C increases by 9.1% on freezing and the latent heat of fusion of ice 80 cal/gm at atmospheric pressure. Calculate the pressure needed to lower the melting point of ice by 1°C. 4

(c) Show that the specific heat at constant volume is related to the second derivative of the Helmholtz free energy. 2

C. U. PHYSICS HONOURS QUESTION PAPERS – 2017

PART – III

FIFTH PAPER – 2017

Full Marks – 100

1. Answer ***any ten*** questions : 2×10

(a) Is the constraint given by $x\dot{x}+y\dot{y}+x\dot{y}+\dot{x}y=k$ (a constant), a holonomic constraint? Give reason to your answer. (b) For a Lagrangian $L(x,\dot{x})=\frac{1}{2}x\dot{x}^2-V(x)$, calculate the Hamiltonian. (c) Water flows through a horizontal tube having variable cross-section. Calculate the increase or decrease in pressure when the velocity of flow changes from 10 cm/sec to 20 cm/sec. (d) In water, can a electron travel faster than light? Explain. (e) Show that the gradient of a scalar function transform as a covariant vector. (f) Prove that if $\frac{v}{c}<<1$, the kinetic energy of a particle will be much less than its rest energy. (g) A simple pendulum with a length of 1 m has a bob with a mass of 0.1 kg. What is its zero point energy? ($\hbar=6.6\times10^{27}$ ergs. sec) (h) Find the constant B which makes $\exp(-ax^2)$ an eigenstate of the operator, $\frac{d^2}{dx^2}-Bx^2$ (i) Show that for a potential $V(\bar{r})=-V(-\bar{r})$, the wave function must be of even or odd parity. (j) Why are the stokes lines brighter than the anti-stokes lines? (k) Under what condition normal Zeeman effect can be observed? (l) What is threshold population inversion?

Group – A

Section – I

Answer ***any two*** questions

2. A particle of mass m moves under the influence of a central potential : $V(r)=-k/r^4$

(a) Show that the motion occurs in a plane. Write the Lagrangian in polar coordinates for the system. Determine all the constants of motion. 4

(b) Plot the effective potential for radial motion of the particle. Give the general condition for a circular orbit. Does the above potential support circular orbits? 1+2+1

(c) In Rutherford scattering of α particles on a thin gold foil, one neglects the effect of the atomic electrons on the α particles, why? 2

3. (a) The point of suspension of a simple pendulum moves simple harmonically along a vertical line. Obtain the Lagrangian of the system. 3

(b) Define Newtonian liquid and non-Newtonian liquid with one example for each case. 3

(c) Starting from Newton's Laws of motion, derive Euler's equation of motion for a liquid. 4

4. (a) What do you mean by generalized coordinates? 2

(b) Show that if the Lagrangian of a closed system does not change when the system is rotated through an infinitesimal angle the angular momentum of the system is conserved. 4

(c) A particle of mass m is constrained to move on a curve $y = f(x)$. Find the Hamiltonian in terms of x and its conjugate momentum and then write down Hamilton's equations.Show that the $\dot{p} = -\partial H/\partial x$ equation reproduces the Euler-Lagrange equation. 4

Section – II

Answer ***any two*** qestions

5. (a) In a frame S the following two events occur

Event 1 : $x_1 = x_0, t_1 = \frac{x_0}{c}$ and $y_1 = z_1 = 0$

Event 2 : $x_2 = 2x_0, t_2 = \frac{x_0}{2c}$ and $y_2 = z_2 = 0$

Find the velocity of the frame S' (w.r. to S) at which these two events occur simultaneously. What is the value of t in S' at which these events are simultaneous? x_0 is a constant and c is speed of light in free space. 4

(b) A space traveller with speed v synchronizes his clock ($t' = 0$) with his earth friend ($t = 0$). The earthman then observes both clocks simultaneously, t directly and t' through a telescope. What does t read when t' reads one hour? 4

(c) A light beam is propagating through a block of glass with index of refraction n. If the block is moving at constant velocity v in the same direction as the beam, what is the speed of light in the block as measured by an observer in the laboratory'? 2

6. (a) Show that Lorentz transformation can be regarded as a rotation of axes (t–x) through an imaginary angle given by $\theta = \tan^1(i\beta)$ where $\beta = \frac{v}{c}$. 3

(b) Show that the ordering of events will remain unaltered in two inertial frames moving with uniform speed relative to each other provided that it is not possible to send any signal with speed greater than the speed of light. 3

(c) Two lumps of clay each of rest mass m_0 move towards each other with equal speed $\frac{3}{5}$c and stick together. What is the mass of the composite lump? 4

7. (a) Establish that $E^2 - p^2c^2$ is a Lorentz invariant quantity, which is equal to $m_0^2c^4$. Symbols have their usual meanings. 4

(b) Show that the scalar product of two four vectors $A_\mu B^\mu$ is invariant under Lorentz transfornation. 3

(c) Show that inner product of the tensors A_k^{ij} and B_r^p is a tensor of rank three. 3

Group – B

Section – I

Answer ***any two*** questions

8. A particle in an infinite square well has as its initial wave function an equal mixture of the first two stationary states :

$$\psi(x,0) = A[\psi_1(x) + \psi_2(x)]$$

Assume $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$ and $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$

(a) Normalize $\psi(x,0)$. 2

(b) Find $\psi(x,t)$ and $|\psi(x,t)|^2$. 3

(c) Compute $\langle x \rangle$. What is the angular frequency and amplitude of the oscillation? 4

(d) Find $\langle H \rangle$. 1

9. (a) An electron confined in a potential well $V(x)=\frac{1}{2}kx^2$, where 'k' is a constant, is subjected to an electric field E along the x-axis. Find the energy eigenvalues. 3

(b) (i) Let ψ_{lm} be an eigenstate of L^2 and L_z with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$ respectively. Show that $\phi=(L_x+iL_y)\psi_{lm}$ is an eigenstate of L^2 and L_z. Determine the eigenvalues. 4

(ii) Show that if $l=0$, the state ψ_{lm} is also an eigenstate of L_x and L_y. 3

10. (a) At time $t=0$ the wave function for a hydrogen atom is,

$$\psi(r,0)=\frac{1}{\sqrt{10}}\left(2\psi_{100}+\psi_{210}+\sqrt{2}\psi_{211}+\sqrt{3}\psi_{21-1}\right)$$

where the subscripts are values of the quantum numbers n, l, m. If one ignores spin and radiative transitions,

What is the probability of finding the system with $l=1$, $m=1$ = as a function of time? 4

(b) Show that $[L_z, x]=i\hbar y$, and $[L_z, y]=i\hbar$. 2

(c) Consider a one-dimensional simple harmonic oscillator moving in a potential $V(x)=\frac{1}{2}m\omega^2x^2$. Given that the ground state wave function is $\psi(x)=\left(\frac{\alpha}{\pi}\right)^{1/4}\exp\left(-\frac{1}{2}\alpha x^2\right)$

(where $\alpha=m/h$) find the ground state energy eigenvalue E_0. Also, calculate the expectation value of $\langle x^2\rangle$. 2+2

Section – II

Answer *any two* questions

11. (a) In a Stern-Gerlach experiment, a beam of silver atoms moving with a velocity of 10^5 *cm/sec* passes through an inhomogeneous magnetic field of gradient 0.5 $Wb/m^2/cm$ along z-axis for a distance of 10 *cm*. What is the separation between the two components of the beam as it comes out of the magnetic field? 4

(b) Consider the $L-S$ coupling scheme for helium atom. Show that (i) $1s^1 2s^1$ configuration leads to the terms $^1S_0$ and $^3S_1$ while (ii) $1s^1 2p^1$ configuration leads to $^1P_1$, $^3P_0$, $^3P_1$ and $^3P_2$. 2+4

12. (a) State Hund's rules for atomic levels. 2

(b) Explain the splitting of atomic spectral lines in presence of a moderate magnetic field by taking spin into account. Hence, explain the splitting of the $(2P_{\frac{3}{2}}\rightarrow 1S_{\frac{1}{2}})$ and $(2P_{\frac{1}{2}}\rightarrow 1S_{\frac{1}{2}})$ lines of the hydrogen spectrum in a moderate magnetic field. Give a neat diagram. 3+3

(c) What is the role played by an optical resonator in a laser system? 2

13. (a) Outline the main features of Raman scattering. 3

(b) Show that in a pure rotational spectrum of a diatomic gas, the spectral lines are equally spaced in frequency. 4

(c) For $2P\rightarrow 1S$ transition in hydrogen atom, the mean spontaneous life-time is 1.66×10^{-18} sec and the frequency of the emitted radiation is 2.4×10^{15} Hz. Calculate the probability of stimulated emission. 3

SIXTH PAPER – 2017
FULL MARKS – 100

Answer ***Question No. 1*** and ***four*** each from **Unit – 11** and **Unit – 12**

1. Answer ***any ten*** of the following : 2×10

(a) A hadron has a quark content ddu. Find the charge and strangeness of this hadron. (b) What is velocity selector in a mass spectrometer? Explain with sketch. (c) What are nuclear isomers? (d) The average nuclear binding energy in the range 30<A<170 is almost constant. Explain why this is so. (e) What is Kurie plot? (f) The maximum energy encountered in β - particle emission from radioactive nuclides is about 4MeV. What is the shortest length of the waves associated with β particles? (g) Sketch the specific heat of a superconductor and normal metal as a function of temperature. (Indicate the critical transition temperature in the graph.) (h) Find the Miller indices for planes with the following sets of intercepts : (i) $(6\bar{a}, 2\bar{b}, 3\bar{c})$ (ii) $(\bar{a}, 2\bar{b}, \infty)$ where $\bar{a}, \bar{b}, \bar{c}$ are lattice vectors. (i) How does a measurement of the potential difference between two points of contact in a metallic sheet in presence of a known magnetic field determines the sign of the charge carriers and their concentration? (j) The Fermi energy (E_F) of silver is 5.5eV. Calculate the fraction of free electrons at room temperature ($\approx 300K$) located within a width of K_BT on either side of E_F. (k) The relative permittivity of argon at 0°C and one atmospheric pressure is 1.000435. Calculate the electronic polarizability of the argon atom. (l) Starting from the dispersion relation $\omega = \omega_m \sin\left(\frac{Ka}{2}\right)$ for linear monatomic chain of length L (= N.a, N is the no. of atoms), obtain and sketch the density of states as a function of ω.

Unit – 11
(Nuclear and particle Physics)

2. (a) Show that an electron confined to a box of nuclear dimension must have an energy more than 20MeV. What will be the order of magnitude of the minimum energy for protons in the above situation? 2+1

(b) A flux of 10^{12} neutrons/m^2 emerges each second from a port in a nuclear reactor. If these neutrons have a Maxwell- Boltzmann energy distribution corresponding to T = 300K;

(i) calculate the average velocity of a neutron. 3

(ii) calculate the density of neutrons in the beam. 4

3. (a) Explain the origin of asymmetry energy in liquid drop model. 3

(b) Show that the law of conservation of angular momentum is not violated in β-decay if the intrinsic spin of the neutrino is $\frac{1}{2}\hbar$. 2

(c) What is the implication of Geiger-Nuttal law in relation with α-decay?

Given that the range in standard air of the α-particles from

radium (half life = 1622 years) is 3.36cm, whereas from polonium (half life = 138 days) this range is 3.85 cm. Calculate the half life of RaC′ for which the α-particlc range is 6.97 ern. 2+3

4. (a) Polonium –212 emits α particles whose kinetic energy is 10.54 MeV. Determine the α-disintegration energy. 3

(b) Find out the spin-parity of the nucleus $^{25}_{13}X$ using extreme single particle shell model. 2

(c) The capture cross-section of ^{59}Co for thermal neutrons is 37b.

(i) What percentage of a beam of thermal neutrons will penetrate a 1.0 mm sheet of ^{59}Co ? The density of ^{59}Co is 8.9×10^3 kg/m^3.

(ii) What is the mean free path of thermal neutrons in ^{59}CO ? 2+2

(d) How one can detect the presence of an excited state of nuclei? 1

5. (a) (i) Using liquid drop model of the nucleus, find the condition for spontaneous fission to occur.

(ii) Explain why a free proton cannot decay through —

$$p \rightarrow n + e^+ + \gamma_e$$ 3+1

(b) Find the minimum kinetic energy in the laboratory system needed by an alpha particle to cause the reaction $^{14}N(\alpha, p)^{17}O$. The masses of ^{14}N, ^4He. ^1H and ^{17}O are respectively 14.00307 amu, 4.00260 amu, 1.00783 amu and 16.99913 amu. 3

(c) Explain the phenomenon of pair production. 3

6. (a) Explain briefly why the cyclotron principle is not used to accelerate protons and the heavier ions to very high energies. 3

(b) An organic quenched GM tube operates at 1 KV and has a wire of diameter of 0.2 mm. The radius of the cathode is 20 mm and the tube has a waranteed life time of 10^9 counts. What is the maximum radial field? How long will the counter last if it is used on the average for 30 hours per week at 3000 counts per minute? 2+2

(c) What do you mean by the recovery time in GM tube? 1

(d) How is the CNO cycle in stars different from the pp chain? 2

7. (a) Which of the following reactions can occur? State the conservation principles violated by the others.

(i) $p+p \rightarrow n+p+\pi^{+}$, (ii) $p+p \rightarrow n+\pi^{0}$,

(iii) $\pi^{+}+p \rightarrow \pi^{+}+p+\pi^{-}+\pi^{0}$. 3

(b) What is color hypothesis? Which type of interaction is supported by this hypothesis? 2+1

(c) Find the missing particles in the following interactions :3

(i) $\mu^{-} \rightarrow e^{-}+\gamma_{e}^{-}+\_\_$, (ii) $\gamma_{e}+n \rightarrow p^{+}\_\_$,

(iii) $e^{+}+e^{-} \rightarrow \mu^{-}+\_\_$.

(d) All resonance particles have very short lifetimes. Why does this suggest they must be hadrons ? 1

Unit – 12
(Solid state Physics)

8. (a) Show that the reciprocal lattice corresponding to a simple cubic lattice is another simple cubic lattice. Find the relation between volumes of the unit cells of the two lattices. 3

(b) Write down Bragg's equation and hence, argue-greater is the angle of diffraction, greater is the accuracy in determining the lattice parameter. 1+2

(c) A diffractometer data of a crystal of an element show peaks at 2θ angles of 44.46Å, 64.7Å, 82Å and 93.22Å. If the wavelength of X-rays used is 1.543Å, assign Miller indices to the peaks and determine the lattice constant. Can you identify the crystal structure? 2+1+1

9. (a) Explain the origin of non-zero value of average energy of degenerate free electrons at T = OK. 2

(b) What is meant by mean free path of free electrons in metal? Calculate the electrical conductivity with mean free path ∧ for a metal with 6×10^{22} conduction electrons per cc in unit of ∧ ohm^{-1} cm^{-1}. 1+3

(c) A uniform copper wire of length 0.5 m and diameter 0.3 mm has a resistance of 0.12 Ω at 293 K. If the thermal conductivity of the specimen at the same temperature is 390 $Wm^{-1}K^{-1}$, calculate the Lorentz number. Compare this value with the theoretical value based on free electron theory. 3+1

10.(a) Mention the different types of bondings in crystalline solids. Describe their properties briefly. 1+2

(b) The potential energy of a system of two atoms is given by

$$U=-\frac{A}{r^{6}}+\frac{B}{r^{12}}.$$

The atoms form a stable bond having bond length 3Å and bond-energy 1.8 eV. Calculate the force required to break the molecule and the critical interatomic distance for which it occurs. 3+1

(c) Find the minimum distance of the Na^{+} and Cl^{-} ions in NaCl crystal. The crystal has FCC interpenetrating lattice structure.

Given : atomic wt. of Na and Cl are 23 and 35.5, density of NaCl is 2.17 gm/cm^3. 3

11. (a) State Bloch's theorem in periodic crystals. 2

(b) In the Krönig-Penney model, the following equation is obtained after simplification :

$$\frac{P}{\alpha a}\sin\alpha a+\cos\alpha a=\cos ka;\ \alpha=\sqrt{\frac{2mE}{\hbar^{2}}};P=\frac{mV_{0}ab}{\hbar^{2}}$$

(i) Obtain the energy band gap at $k=\pi/a$ in the limit $V_{0}ab << \frac{\hbar^{2}}{m}$.

(ii) What is the energy of the lowest band at k = 0 in the limit P << 1? 2+2

(c) Find the expression for effective mass of an electron in a lattice. What happens to this mass close to the edges of the Brillouin zones and why? 2+2

12. (a) Suppose a paramagnetic atom having permanent moment $\bar{\mu}$ with a given resultant quantum number $\bar{J}$, is placed in a uniform magnetic field $\bar{B}$. Obtain an expression of the magnetization as a function of B and temperature T. Hence, obtain Curie's law in the appropriate limit. 4+2

(b) Sketch the spontaneous magnetization of a ferromagnet as a function of temperature. Indicate the universal feature associated with the graph. 1+1

(c) Atomic weight and density of iron are 55.847 and 7.87×10^3 kg/m^3 respectively. If iron has a magnetic moment of 2.2 Bohr magneton, determine its spontaneous magnetization. 2

13. (a) Explain briefly the Meissner effect with a suitable diagram. 2 + 1

(b) Calculate the wavelength of the photon which will be required to destroy the superconductivity in Aluminium having critical transition temperature 1.2 K. In which region of electromagnetic spectrum does it belong? 1 + 1

(c) What are the main assumptions in Debye model? How does it differ from Einstein's model? Find the expression for number of modes of vibrations in the range γ to $\gamma + d\gamma$ in a cubic solid having N atoms. 1+1+3

SEVENTH PAPER – 2017

(Group – A)

FULL MARKS – 50

Answer ***Question No. 1*** and ***any four*** from the rest

1. Answer ***any five*** of the following : 2×5

(a) By evaluating the volume of the relevant region of its phase space, show that the number of microstates available to a rigid rotator with angular momentum less than M is $(2\pi M / h)^2$.

(b) State the theorem of equipartition of energy with particular emphasis on the form of the system's Hamiltonian.

(c) Sketch the Fermi distribution function for three temperatures $T_1 > T_2 > T_3 = 0$ on the same graph.

(d) Explain how Maxwell modified the Ampere's law.

(e) The non-zero components of the electric and magnetic vectors for an electromagnetic field are given by $E_y = E_0 e^{i(kx - \omega t + \phi)}$, $B_z = B_0 e^{i(kx - \omega t)}$ respectively. Obtain the value of ϕ, when $0 \le \phi < 2\pi$.

(f) Find out the pressure exerted by a radiation of power 1kW/cm^2 on a totally absorbing surface.

2. (a) Derive the partition function of a system of classical monatomic ideal gas. How do you show the extensive property of the entropy from this partition function?

(b) A system has two non-degenerate energy states E_1 and E_2, with populations n_1 and n_2, respectively ($n_1, n_2 >> 1$). The system is in contact with a heat bath at temperature T. Show, from Boltzmann's statistical definition of entropy, the change in entropy when one particle goes from state 1 to state 2 is given by $k\ln(n_2 / n_1)$ where k is the Boltzmann constant.

(c) A system has three non-degenerate energy states given by ε_1=OK, ε_2/k = 200K and ε_3/k = 300K . Find the average energy $\langle\varepsilon\rangle$ and dispersion $\sigma = \sqrt{\langle\varepsilon^2\rangle - \langle\varepsilon\rangle^2}$ for the system if the temperature of the system is 250K. (3+3)+2+2

3. Find out the number of ways in which n identical bosons may be distributed among g energy levels.

Obtain an expression for electronic specific heat for metals at low temperature.

Á spherical black body of radius 1 cm is enclosed in an evacuated chamber. If the chamber is at a temperature 300K, find out the amount of heat that must be supplied per second to the black body to keep it at a temperature 1000K. Neglect conduction of heat, ($\sigma = 5.67 \times 10^{-8}$ W/m^2/K^4). 2+4+1+3

4. (a) With proper explanation, derive the condition of validity of classical statistical mechanics in terms of the temperature and density of particles.

(b) Show that the Fermi momentum is proportional to the cube root of the density of free fermions at zero temperature in three dimensions. 5+5

5. Starting from Maxwell's equation, show that any initial charge density in a conductor dissipates in a characteristic time.

Find the average energy density for a plane monochromatic wave.

Show that Poynting theorem predicts Joule heating in a wire. 3+3+4

6. A current I is made to increase in the windings of a long solenoid very slowly. The solenoid has radius R, length L and 'n' turns per unit length.

(a) Write down the expression for energy stored U, in the solenoid at any given instant.

(b) Find out the time rate of change of U in terms of $\frac{dI}{dt}$.

(c) Use Faraday's law to find the induced electric field.

(d) Find out the instantaneous Poynting vector.

(e) Calculate its flux through the Lateral surface (i.e. the cylindrical surface, not the edges) and find its relation with $\frac{dU}{dt}$.

2+2+2+2 +2

7. An electromagnetic wave is incident at the surface of two linear homogeneous dielectrics. Write down the boundary conditions at the surface. Find out the ratio of the electric field intensities for normal incidence. Find out the conditions under which there is a phase reversal for the reflected wave.

Starting from Maxwell's equations derive Kirchhoff's voltage and Current Laws.

2+3+2+3