Spot & Forward rates and IRPT

(Economics Hons Semester 5: Financial Economics)

The Interest Rate Parity Theorem (IRPT) is a fundamental law of international finance. We know that there are interest rate differentials between countries. Say, an Indian Bond yields 10% and a US bond yields 1%. Why wouldn't capital flow to India from USA until this differential disappeared? Assuming that there are **no government restrictions to the international flow of capital** or **zero transaction costs**, where is the barrier to complete interest rate equalisation?

The barrier that prevents US capital to fly to India is currency risk. Once dollars are exchanged for the Indian Rupees, there is no guarantee that the Rupee will not depreciate against the dollar. There is, however, one way to guarantee a conversion rate between the dollars and the Rupee: a trader can use a **forward foreign currency contract**.

Forward foreign currency contracts eliminate currency risk. A forward foreign currency contract allows a trader to compare domestic returns with foreign returns translated into the domestic currency, without facing currency risk. **Arbitrage** (buying cheap and selling dear) will ensure that both known returns, expressed in the same currency, are equal. That is, world interest rates are linked together through the currency markets. The IRPT is a reflection of this relation:

If the interest rate on a foreign currency is different from that of the domestic currency, the forward exchange rate will have to be different from the spot exchange rate by a sufficient amount to make profitable arbitrage impossible.

A Covered interest arbitrage: Covered interest arbitrage is the activity that forces the IRPT to hold. Assume that there are no barriers to the free movement of capital across international borders –i.e., there is perfect capital mobility. Consider the following notation:

i_d = domestic nominal risk-free interest rate for T days.

i_f = foreign nominal risk-free interest rate for T days.

S_t = time t spot rate (direct quote: units of domestic currency per unit of foreign currency).

$F_{t,T}$ = forward rate at time t, for delivery at date T.

Now, consider the following strategy:

(1) At time 0, we borrow from a foreign bank one unit of a foreign currency for T days. At time T, we should pay the foreign bank (1+ i_f x T/365) units of the foreign currency.

(2) At time 0, we exchange the unit of foreign currency for domestic currency, that is, we get S_0 units of domestic currency.

(3) At time 0, we deposit S_0 units of domestic currency in a domestic bank for T days. At time T, we should receive from the domestic bank S_0 (1+ i_d x T/365) units of domestic currency.

(4) At time 0, we also enter into a T-day forward contract to buy foreign (sell domestic currency) at a pre-specified exchange rate $F_{t,T}$ .At time T, we exchange the S_0 (1+ i_d x T/365) units of domestic currency for foreign currency, using the pre-specified exchange rate in the forward contract. That is, we get

S_0 (1+ i_d x T/365) / ($F_{t,T}$) units of foreign currency. This strategy will not be profitable if at time T, what we receive in units of foreign currency is equal to what we have to pay in units of foreign currency. Since arbitrageurs will be searching for an opportunity to make a risk-free profit, arbitrage will ensure that

$$S_0 (1+ i_d \times T/365) / (F_{t,T}) = (1+ i_f \times T/365).$$

Solving for ($F_{t,T}$), we obtain the following expression for the IRPT:

$$(F_{t,T}) = S_0 (1+ i_d \times T/365)/ (1 + i_f \times T/365).$$

Behind the covered arbitrage strategy steps (1) to (4), we have implicitly assumed:

(1) Funding is available. That is, step (1) can be executed.

(2) Free capital mobility. No barriers to international capital flow – i.e., step (2) and later (4) can be implemented.

(3) No default/country risk. That is, steps (3) and (4) are safe.

(4) Absence of significant frictions. Typical examples: transaction costs & taxes. Small transactions costs are OK, as long as they do not impede arbitrage.

(5) We are also implicitly assuming that the forward contract for the desired maturity T is available. This may not be true. In general, the forward market is liquid for short maturities (up to 1 year). For many currencies, say from emerging market, the forward market may be liquid for much shorter maturities (up to 30 days).