# ADVANCED BUSINESS MATHEMATICS - HONOURS 

Paper: DSE-5.1A
(Module - II)
Full Marks: 40
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words
as far as practicable.

Answer any four questions.
$10 \times 4=40$

1. (a) Evaluate : $\log _{x \rightarrow 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$.
(b) The total cost $C$ of producing $x$ items is given by $C=\left\{\begin{array}{c}100+5 x, 0 \leq x \leq 500 \\ 2000+4 x, 500<x \leq 2000\end{array}\right.$ Show that $C$ is discontinuous at $x=500$.
2. (a) If $A-2 B=\left[\begin{array}{rr}-7 & 7 \\ 4 & -8\end{array}\right]$ and $A-3 B=\left[\begin{array}{rr}-11 & 9 \\ 4 & -13\end{array}\right]$, find the matrices $A$ and $B$.
(b) Prove that $\left|\begin{array}{ccc}x & y & z \\ x^{2} & y^{2} & z^{2} \\ x^{3} & y^{3} & z^{3}\end{array}\right|=x y z(x-y)(y-z)(z-x)$.
3. (a) Evaluate : $\int \frac{(4 x-3)^{3}}{x^{2}} d x$.
(b) Find $\frac{d^{2} y}{d x^{2}}$, if $y^{3}+3 a x^{2}+x^{3}=0$.
(2)
4. (a) If $y=\log \left(x+\sqrt{1+x^{2}}\right)$, then show that $\left(1+x^{2}\right) y_{2}+x y_{1}=0$.
(b) A firm produces $x$ tonnes of output at a total cost Rs. $R$ where $R=\frac{1}{10} x^{3}-5 x^{2}+10 x+5$. Find at what level of output, average cost be minimum and what level will it be.
5. (a) Find the area included between $y^{2}=9 x$ and $y=x$.
(b) The price $p$ and quantity $q$ of a commodity are related by $q=32-4 p-p^{2}$; find the marginal revenue when $p=3$.
6. (a) Evaluate : $\int \frac{d x}{\sqrt{x+1}-\sqrt{x-1}}$.
(b) Evaluate : $\int \frac{5 x+2}{(x-2)(x-3)} d x$.
7. (a) Solve by Cramer's Rule the following set of equations :
$2 x+3 y-z=9 ; \quad x+y+z=9 ; \quad 3 x-y-z=-1$.
(b) Verify that the matrix $A=\frac{1}{3}\left[\begin{array}{rrr}-1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1\end{array}\right]$ is an orthogonal matrix.
8. Find the inverse of $\left[\begin{array}{rrr}1 & 1 & -2 \\ -2 & 1 & -2 \\ 1 & 0 & 2\end{array}\right]$ and hence solve the following system of equations

$$
\begin{align*}
& x+y-2 z=4 ;-2 x+y-2 z=1 ; \quad x+2 z=3 .  \tag{10}\\
& 8
\end{align*}
$$

