City College

Internal Examination 2020

Physics (Hons.) CBCS Semester 4

Paper: CC-8: Mathematical Physics III

Time: 2 Hours; Full Marks: 50

Group A

1. Answer any five questions from the following: [$5 \times 2 = 10$]

(a) An unbiased dice is thrown three times successively. What is the probability that the numbers of dots on the uppermost surface add up to 16?

(b) Find the Fourier transform of $\delta(x)$.

(c) Does the density of an object change as its speed increases? If yes, by what factor?

(d) Show that the 4-dimensional volume element $dxdydzdt$ is invariant under Lorentz transformation.

(e) For what value(s) of z is $f(z) = \frac{1}{z-i}$ not defined? Writing $f(z) = u + iv$, find u and v for such value(s) of z.

(f) Find the singularities of the function

$$f(z) = \frac{1}{1-z} - \frac{1}{1+z}.$$

Group B

Answer any five questions [$5 \times 5 = 25$]

2. (a) A random number generator outputs $+1$ or -1 with equal probability every time it is run. After it is run six times, what is the probability that the sum of the answers generated is zero? Assume that the individual runs are independent of each other. (b) You receive on average five e-mails per day during a 365-days year. How many days in a year on average you do not receive any emails? [$3 + 2 = 5$]

3. Find the Fourier transform $F(k)$ of the function $f(x) = Ne^{-\alpha x^2}$, where N and α are constants. Calculate the standard deviation of both $F(k)$ and $f(x)$. Plot $F(k)$ against k and $f(x)$ against x. [$3 + 1 + 1 = 5$]

4. Consider a Lorentz transformation in a 2-dimensional space-time given by

$$\begin{aligned} ct' &= \gamma(ct - \beta x) \\ x' &= \gamma(-\beta ct + x), \end{aligned}$$

where $\gamma = (1-\beta^2)^{-\frac{1}{2}}$ and $\beta = \frac{v}{c}$. Write down the matrix $\Lambda(\beta)$ of the above transformation and find (i) $\Lambda(0)$ and (ii) determinant of $\Lambda(\beta)$. Check if $\Lambda(\beta)$ is orthogonal. [$1 + 1 + 1 + 2 = 5$]

5. (a) Two particles, each of rest mass m, collide head-on and stick together. Before collision, the speed of each mass was 0.6 times the speed of light in free space. What is the mass of the final entity? (b) In an observer's rest frame, a particle is moving towards the observer with an energy E and momentum P. If c denotes the velocity of light in vacuum, what is the energy of the particle in another frame moving in the same direction as the particle with a constant velocity v? $[3+2=5]$

6. (a) Show that if $f(z)$ is an analytic function, then $\frac{df}{dz^*} = 0$. (b) For $f(z) = u + iv$, given that $u = x^2 - y^2$, determine $f(z)$ up to an additive constant using Cauchy-Riemann conditions. $[3+2=5]$

7. (a) Evaluate

$$\oint_{\mathcal{C}} \frac{e^z}{z^2+1} dz$$

where $\mathcal{C}$ is a circle of unit radius centered at (i) $z = i$, (ii) $z = -i$. (b) Find the residue of $f(z) = \frac{\tan z}{z^2}$ at $z = 0$. $[3+2=5]$

Group C

Answer any five questions $[5 \times 3 = 15]$

8. A function is defined as

$$f(x) = \begin{cases} x, & \text{for} \quad 0 < x < +1 \\ -x, & \text{for} \quad -1 < x < 0 \end{cases}$$

and $f(x+2) = f(x)$. Plot the function. Extend it suitably over the interval -4 to $+4$ and plot the same.

9. Write down the Fourier series expansion of the function $f(x) = \sin^2 x$. A periodic function can be expressed in the form

$$f(x) = \sum_{n=0}^{\infty} \left[a_{2n} \cos x + a_{2n+1} \sin x\right].$$

If $f(x) = |\sin x|$, find a_{2n+1}.

10. In the equation

$$\frac{\partial u}{\partial t} = \frac{1}{\alpha^2} \frac{\partial^2 u}{\partial x^2}$$

find the dimension of the constant α in terms of length and time. Is the equation linear?

11. Write a program to evaluate the improper integral

$$\mathrm{I} = \int_1^{\infty} \frac{e^x}{\sqrt{x}} dx.$$

12. Mention any two type of boundary conditions that may be enforced to solve a partial differential equation.

13. Consider the ordinary differential equation

$$\frac{d^4y}{dx^4} + \frac{dy}{dx} - \rho(x) = 0.$$

How many boundary conditions are required to solve the equation? Does the superposition principle hold for solutions of the equation?

Answer scripts must be emailed to ***sem4hcityphysics@gmail.com*** *within 15 minutes of the end of the examination.*