1. State and explain Liouville's theorem.

2. What is ergodic hypothesis?

3. A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. What is the entropy of the system?

4. Consider a system of three fermions which can occupy any of the four available energy states with equal probability. Find the entropy of the system.

5. Consider a 1-dimensional Fermi gas confined along a line of length ℓ. Show that the Fermi momentum is

$$p_{\text{F}} = \pi\hbar\frac{N}{2\ell}.$$

6. Consider a gas contained in volume V at temperature T. The gas is composed of N distinguishable particles of zero rest mass, so that energy E and momentum p of the particle are related by $E = pc$. Show that the partition function of the system is

$$\mathcal{Z} = \left[\left(\frac{kT}{c}\right)^3 \left(\frac{8\pi V}{h^3}\right) \right]^N$$

7. Electromagnetic radiation in a box of volume V can be treated as a non-interacting ideal Bose gas of photons. If the cavity also contains atoms capable of absorbing and emitting photons, the number of photons in the cavity is not definite. Find the average number of photons in the box. You may need the following result:

$$\int_0^\infty \frac{x^2}{e^x - 1}dx \sim 2.4.$$

8. A 1-dimensional harmonic oscillator has an infinite series of equally spaced energy states, $\epsilon_n = n\hbar\omega$, where n is a non-negative integer. Show that the free energy of the system is $F = kT\ln\left(1 - e^{-\frac{\hbar\omega}{kT}}\right)$.

9. For photons, the grand canonical partition function reduces to the canonical partition function. Explain.

10. Consider N spins in a chain which can be modeled using the 1-dimensional Ising model

$$H = -J \sum_{n=1}^{N-1} s_n s_{n+1}$$

where the spin s_n has values $s_n = \pm 1$. Find the partition function.

11. What do you mean by a pure state and a mixed state?

12. Show that $\text{Tr}\rho = 1$, where ρ is the density matrix.

13. Show that the density matrix satisfies $\rho^2 = \rho$.

14. Does a Bose gas exert any pressure at absolute zero?

15. In how many ways can you distribute N bosons in two energy levels?

*Answer scripts must be emailed to **sem6hcityphysics@gmail.com** within 15 minutes of the end of the examination.*