CALCUTTA UNIVERSITY

B.Sc. MATHEMATICS HONS. QUES. PAPERS (PART – I & II) – 2017

PART – I

FIRST PAPER – 2017

(Module – I)

Full Marks – 50

Group – A : [Marks – 35]

Answer *any seven* questions

1. State the "First Principle of Mathematical Induction". Using this principle show that $10^{n+1} + 10^n + 1$ is divisible by 3 for all positive integers n. 1+4

2. (a) Prove that $\phi(3n) = 3\phi(n)$ if and only if 3 is a divisor of n. 3

(b) If p be a prime and k be a positive integer, prove that

$$\phi(p^k) = p^k\left(1 - \frac{1}{p}\right).$$ 2

3. (a) Find the remainder when $1! + 2! + 3! + \ldots + 50!$ is divided by 30. 2

(b) Prove that the product of any m consequtive integers is divisible by m. 3

4. Given integers a and b with $b > 0$, prove that there exist unique integers q and r such that $a = bq + r$, $0 \le r < b$. 5

5. If $\log \sin(\theta + i\phi) = \alpha + i\beta$, prove that $2\cos 2\theta = e^{2\phi} + e^{-2\phi} - 4e^{2\alpha}$ and $\cos(\theta - \beta) = e^{2\phi}\cos(\theta + \beta)$. 5

6. (a) Show that the values of i^i are all real. 1

(b) Prove that $\sin\left(i \log \frac{a - ib}{a + ib}\right) = \frac{2ab}{a^2 + b^2}$. 2

(c) Find the general solution $\sinh z = 2$. 2

7. (a) If $a_1, a_2, a_3, \ldots\ldots, a_n$ be n positive numbers and $a_1 + a_2 + a_3 + \ldots\ldots + a_n = s$, then show that

$$\frac{s}{s-a_1} + \frac{s}{s-a_2} + \frac{s}{s-a_3} + \ldots\ldots + \frac{s}{s-a_n} \ge \frac{n^2}{n-1}.$$ 3

(b) If a, b, c be all positive rational numbers, not all are equal, then using generalised m-th power theorem show that

$$(a^2b+b^2c+c^2a)(ab+bc+ca)>abc(a+b+c)^2. \quad 2$$

8. If $a_1, a_2, \ldots, a_n$ are positive rational numbers and not all equal and S is their sum, show that

$$\left(\frac{S}{a_1}-1\right)^{a_1}\left(\frac{S}{a_2}-1\right)^{a_2}\cdots\left(\frac{S}{a_n}-1\right)^{a_n}<(n-1)^s. \quad 5$$

9. $\phi(x)=0$ be the equation whose roots are the squares of the roots of the cubic $f(x)\equiv x^3-ax^2+bx-1=0$. If $f(x)=0$ are identical, show that either $a=b=0$ or $a=b=3$ or a, b are the roots of $t^2+t+2=0$. 5

10. Show that if the roots of the equation $x^4+x^3-4x^2-3x+3=0$ are increased by 2 then the transformed equation becomes a reciprocal equation. Solve the reciprocal equation and hence obtain the solution of the given equation. 2+2+1

11. (a) Show that the special roots of the equation $x^9-1=0$ are the roots of the equation $x^6+x^3+1=0$. 2

(b) Solve the equation $x^3-18x-35=0$ by Cardan's method. 3

12. (a) If the equation $x^n-px^2+r=0$ has two equal roots, show that

$$n^n r^{n-2}=4p^n(n-2)^{n-2}. \quad 3$$

(b) If $x^4-14x^2+24x-k=0$ has four real and unequal roots, prove that k must lie between 8 and 11. 2

13. If $\alpha,\beta,\lambda,\delta$ be the roots of the equation $x^4+px^3+qx^2+rx+s=0$, prove that $(\alpha\beta+\gamma\delta)(\beta\gamma+\alpha\delta)(\gamma\alpha+\beta\delta)=r^2-4qs+p^2s$. 5

Group – B : [Marks – 15]

Answer *any three* questions

14. (a) For any two subsets A and B of some universal set U, prove that $(A\Delta B)'=(A\cup B)'\cup(A\cap B)$. 2

(b) Let $\rho=\{(a,b)\in z\times x : 3a+4b=7n$, for some $n\in z\}$. Show that ρ is an equivalence relation. 3

15. (a) For a finite set S, if $f: S\to S$ be injective then show that f is surjective. 2

(b) Let $f: X\to Y$ be a mapping. Then prove that $f(A-B)\subseteq f(A)-f(B)$. 3

16. Prove that any finite semigroup in which both cancellation laws hold is a group. 5

17. (a) Let G be a group such that for any two elements $a,b\in G$, $(ab)^3=a^3b^3$ and $(ab)^5=a^5b^5$. Prove that G is commutative. 3

(b) Let $(G,\circ)$ be a group and $a,b\in G$. If $O(a)=3$ and $a\circ b\circ a^{-1}=b^2$, find $O(b)$ if $b\neq e$. 2

18. (a) Show by an example that the union of two subgroups of a group is not necessarily a subgroup of that group. 2

(b) Let G be an Abelian group and $H=\{a\in G : O(a) \text{ is finite}\}$. Prove that H is a subgroup of G. 3

FIRST PAPER – 2017

(Module - II)

Full Marks - 50

Group – A : Full Marks – 20

Answer *Question No.1* and *any two* from the rest

1. Answer *any one* question :

(a) (i) If the triangle formed by the straight lines $ax^2+2hxy+by^2=0$, (where $a+b\neq 0$) and the straight line $lx+my=1$ is right-angled, then show that $al^2+2hlm+bm^2=0$. 2

(ii) If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two straight lines, then prove that the product of the lengths of the perpendiculars from the origin on those lines is

$$\frac{|c|}{\sqrt{(a-b)^2+4h^2}}. \quad 6$$

(b) (i) Transform the equation $3x^2+8xy+3y^2-2x+2y-6=0$ referred to new axes through $(-1,1)$ rotated through an angle $\pi/4$. 2

(ii) Reduce the equation $4x^2+4xy+y^2-4x-2y+a=0$ to its canonical form and determine the type of the conic represented by it, for different values of a. 6

2. Prove that the locus of the point of intersection of pair of perpendicular tangents to the conic $\frac{l}{r}=1+e\cos\theta$ is $\left(e^2-1\right)r^2-2ler\cos\theta+2l^2=0$. 6

3. Two lines are drawn at right angles, one being a tangent to $y^2=4ax$ and the other being tangent to $x^2=4by$. Show that the locus of their point of intersection is the curve $\left(x^2+y^2\right)(ax+by)+(bx-ay)^2=0$. 6

4. Chords of the ellipse $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$ touch the director circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Find the locus of their poles. 6

5. Show that the equation of the line joining the feet of the perpendiculars from the point $(d, 0)$ on the lines $ax^2+2hxy+by^2=0$ is $(a-b)x+2hy+bd=0$. 6

Group - B : Full Marks - 15

Answer ***Question No.6*** and ***any two*** from the rest

6. Prove that the straight lines whose direction cosines are given by $2l+2m-n=0$ and $mn+nl+lm=0$ are at right angles. 3

Or, Show that a first degree equation in x, y, z always represents a plane. 3

7. A variable plane has intercepts on the coordinate axes the sum of whose squares is k^2. Show that the locus of the foot of the perpendicular from the origin to the plane is

$$\left(x^2+y^2+z^2\right)^2\left(x^{-2}+y^{-2}+z^{-2}\right)=k^2.$$ 6

8. Show that the length of the shortest distance between the line $z=x\tan\alpha$, $y=0$ and any tangent to the ellipse $x^2\sin^2\alpha+y^2=a^2$, $z=0$ is constant. 6

9. (i) A variable line intersects the lines $y=0, z=c$; $x=0, z=-c$ and is parallel to the plane $lx+my+nz=p$. Show that the surface generated by it is $lx(z-c)+my(z+c)+n(z^2-c^2)=0$. 4

(ii) Prove that, if the planes $ax+hy+gz=0$, $hx+by+fz=0$, $gx+fy+cz=0$ have a common line, then $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$. 2

10. If θ be the angle between the two lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2, show that the direction cosines of their angular bisectors are

$$\frac{l_1+l_2}{2\cos\frac{\theta}{2}}, \frac{m_1+m_2}{2\cos\frac{\theta}{2}}, \frac{n_1+n_2}{2\cos\frac{\theta}{2}} \text{ and } \frac{l_1-l_2}{2\sin\frac{\theta}{2}}, \frac{m_1-m_2}{2\sin\frac{\theta}{2}}, \frac{n_1-n_2}{2\sin\frac{\theta}{2}}.$$ 6

Group - C : Full Marks - 15

Answer ***any three*** questions

11. (a) Prove by vector method that the external bisector of any angle of a triangle divides the opposite side externally in the ratio of the two sides containing the angle. 5

12. In any triangle ABC, if $\overline{BC}=\bar{a}$, $\overline{CA}=\bar{b}$, $\overline{AB}=\bar{c}$ prove that $c^2=a^2+b^2+2\bar{a}\bar{b}$ where $a=|\bar{a}|$, $b=|\bar{b}|$, $c=|\bar{c}|$ and $\Delta=\frac{1}{2}|\bar{a}\times\bar{b}|$ where Δ is the area of the triangle ABC.

Using the above two relations, prove that $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{1}{2}(a+b+c)$. 1+2+2

13. (i) Prove that

$$\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\left[\vec{a}\ \vec{c}\ \vec{d}\right]\vec{b}-\left[\vec{b}\ \vec{c}\ \vec{d}\right]\vec{a}=\left[\vec{a}\ \vec{b}\ \vec{d}\right]\vec{c}-\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{d}$$

Hence express $\vec{d}$ in terms of the non-coplanar vector $\vec{a}, \vec{b}, \vec{c}$ 2+1

(ii) If $\hat{a}, \hat{b}, \hat{c}$ be three unit vectors such that $\hat{a}\times(\hat{b}\times\hat{c})=\frac{\sqrt{3}}{2}\hat{b}$, find the angles which $\hat{a}$ makes with $\hat{b}$ and $\hat{c}$ where $\hat{b}$ and $\hat{c}$ are non-parallel. 2

14. (i) If the straight lines $\vec{r}=\vec{a}+t(\vec{b}\times\vec{c})$ and $\vec{r}=\vec{b}+s(\vec{c}\times\vec{a})$

intersect then show that $\vec{a}.\vec{c} = \vec{b}.\vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors. 3

(ii) If the vectors $\vec{a}, \vec{b}, \vec{c}$ be linearly independent, then show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$, $-\vec{b} + 2\vec{c}$ are linearly dependent. 2

15. (i) Determine the value of λ for which the vector equation $\vec{a} - (\vec{x} \times \vec{b}) = \lambda\vec{b}$ is solvable and then solve it. 3

(ii) A force $\vec{F}$ of magnitude 10 units acts along the line $\frac{x-2}{5} = \frac{y-1}{4} = \frac{z-3}{3}$. Find the torque of the force $\vec{F}$ about z-axis. 2

SECOND PAPER – 2017
(Module - III)
Full Marks - 50

IR, Q, IN denote the sets of real numbers, rational numbers and natural numbers respectively.

Group - A (Marks - 40)

Answer ***any four*** questions

1. (a) State and prove Archimedean property of real numbers. 1+3

(b) Find Sup A and Inf A where

$$A = \left\{ x \in \mathbb{R} : \sin\frac{1}{x} = 0 \right\}.$$ 3

(c) Let f : [0, 1] → IR be a positive valued continuous function on [0, 1]. Let g : [0, 1] → IR be defined by

$$g(x) = \begin{cases} f(x) & \text{if } x \in Q \cap [0,1] \\ 0 & \text{if } x \in [0,1] - Q \end{cases}$$

Find the closure of the set $S = \{x \in [0,1] : g$ is discontinuous at $x\}$. 3

2. (a) Show that [0, 1] is not enumerable. 4

(b) Prove or disprove : If S is an infinite bounded above subset of IR, then S has a limit point. 2

(c) Construct an example of a closed and bounded subset of IR having enumerable derived set. 4

3. (a) Show that arbitrary union of open subsets of IR is an open set. What can you say about arbitrary intersection of open sets ? 3+2

(b) Prove or diprove : Every bounded infinite subwet of IR has an interior point. 2

(c) Let S be a closed and bounded subset of IR. If each point of S is an isolated point of S, then show that S is finite set. 3

4. (a) Let $\{x_n\}$ and $\{y_n\}$ be two sequences of real numbers such that $\lim_{n\to\infty} x_n = \ell$, $\lim_{n\to\infty} y_n = m(\neq 0)$ and $y_n \neq 0\ \forall n \in \mathbb{N}$. Show that $\lim_{n\to\infty} \frac{x_n}{y_n} = \frac{\ell}{m}$. 4

(b) If a > 0, prove that $\{a^{1/n} - 1\}$ is a null sequence. 3

(c) Prove or disprove : If $\{x_n\}$ and $\{y_n\}$ be two sequences of real numbers such that $\lim_{n\to\infty} x_n = 0$, then $\lim_{n\to\infty} (x_n y_n) = 0$. 3

5. (a) Prove that every sequence of real numbers has a monotone subsequence. 4

(b) If for a sequence $\{x_n\}$ of real numbers,

$$\lim_{n\to\infty} x_{3n-2} = \lim_{n\to\infty} x_{3n-1} = \lim_{n\to\infty} x_{3n} = \ell(\in \mathbb{R}),$$

then prove that $\{x_n\}$ is convergent. 3

(c) Find $\overline{\lim}\, a_n$ and $\underline{\lim}\, a_n$ if $a_n = \left(2\cos\frac{n\pi}{2}\right)^{(-1)^{n+1}}$. 3

6. (a) If f : [a, b] → IR is continuous on [a, b], prove that f is bounded on [a, b]. 3

(b) Let f : [0, 1] → IR be a continuous function on [0, 1] and c ∈ (0, 1) be such that f(c) = c. Show that there exists a neighbourhood N(c) of c such that f(x) > 0 ∀ x ∈ N(c). 3

(c) Correct or justify : There exists a monotonic function defined on [0, 1] such that the function is discontinuous at every irrational point in [0, 1]. 2

(d) Prove or disprove : If f, g : $\mathbb{R} \to \mathbb{R}$ be two functions such that both the functions are discontinuous at a point $c \in \mathbb{R}$, then f.g is also discontinuous at c. 2

7. (a) Let $f : (a, b) \to \mathbb{R}$ be a monotonically decreasing function and $c \in (a, b)$. Prove that $\lim\limits_{x \to c+} f(x)$ exists finitely. 3

(b) If $f : R \to \mathbb{R}$ is uniformly continuous on $\mathbb{R}$ and $\{x_n\}$ is a Cauchy sequence, then prove that $\{f(x_n)\}$ is also a Cauchy sequence.

Test uniform continuity of $\sin\frac{1}{x}$ on $(0,\infty)$. 2+2

(c) Prove or disprove : If $f : \left[0,\frac{\pi}{2}\right] \to [0,1]$ is continuous on $\left[0,\frac{\pi}{2}\right]$, then there exists a point $c \in \left[0,\frac{\pi}{2}\right]$ such that f(c) = sin c. 3

Group - B (Marks - 10)

8. Answer *any two* questions :

(a) If $I_{m,n} = \int \sin^m x \cos^n x \, dx$, where m, n are positive integers, then prove that

$$I_{m,n} = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} I_{m,n-2}.$$

Hence find a reduction formula for

$$J_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x \, dx.$$ 4+1

(b) Evalutae : $\int_0^{\pi} \frac{dx}{(5+4\cos x)^2}$. 5

(c) Evaluate : $\int \frac{5x+7\sqrt{1-x^2}}{3(1-x^2)-2x\sqrt{1-x^2}} dx$. 5

(d) Show that

$$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{n^2+2.1^2}+\frac{n}{n^2+2.2^2}+\ldots+\frac{n}{n^2+2(n-1)^2}\right]=\frac{1}{\sqrt{2}}\tan\sqrt{2}$$ 5

SECOND PAPER – 2017

(Module - IV)

Full Marks - 50

Group - A : (Linear Algebra - I) : (Marks - 35)

Answer ***Question No.1*** and ***any three*** from the rest

1. Answer ***any one*** of the following :

(a) Prove that

$$\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$$

. where $2s = a + b + c$. 5

(b) (i) B is nonsingular matrix such that the sum of the elements in each column is $k\ (k \neq 0)$. Prove that the sum of the elements in each column of B^{-1} is k^{-1}. 3

(ii) P is an $n \times n$ real otthogonal matrix with det $P = -1$. Prove that $P + I_n$ is a singular matrix. 2

2. (a) (i) Let U and W be two subspaces of a vector space V over a field F and $U + W = \{u + w : u \in U, w \in W\}$. Prove that $U + W$ is the smallest subspace of V containing the subspaces U and W. 4

(ii) Let $\{\alpha, \beta, \gamma\}$ be a linearly independent set in a vector space V over a field F. Is $\{\alpha-\beta, \beta-\gamma, \gamma-\alpha\}$ linearly independent ? Justify your answer. 1

(b) (i) For what real values of k does the set S form a basis of $\mathbb{R}^3$ where $S = \{(k,0,1),(1,k+1,1),(1,1,1)\}$? 3

(ii) In the vector space $\mathbb{R}^3$, $\alpha = (1,2,1)$, $\beta = (3,1,5)$ and

$\gamma = (3,-4,7))$. Show that $L(S) = L(T)$ where $S = \{\alpha, \beta\}$ and $T = \{\alpha, \beta, \gamma\}$. 2

3. (a) Prove that there exists a basis for each finite dimensional vector space. 5

(b) (i) Find a basis for the vector space $\mathbb{R}^3$ that contains the vectors $(1,2,1)$ and $(3,6,2))$. 3

(ii) Find the dimension of the subspace

$$S = \{(x,y,z) \in \mathbb{R}^3 / 2x + y - z = 0\} \text{ of } \mathbb{R}^3.$$ 2

4. (a) Find a basis of the row space for the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$ 5

(b) Using congruent operation, reduce the quadratic

$$x^2 + 4y^2 + z^2 - 4yz + 2zx - 4xy$$

into its normal form and hence find its rank and signature. 3+1+1

5. (a) Find an orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix where $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$. 5

(b) (i) If M is an orthogonal matrix and $|A| = -1$, then show that (-1) is an eigenvalue of A. 3

(ii) Find the eigenvalues of the complex matrix $\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$. 2

6. (a) Verify Cayley-Hamilton theorem for the matrix

$$M = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{bmatrix}$$ and utilize it to find M^{-1}. 5

(b) (i) If α, β be any two vectors in a Euclidean space V, then show that $\|\alpha+\beta\|^2 - \|\alpha-\beta\|^2 = 4\langle\alpha,\beta\rangle$. 2

(ii) For any two vectors α, β in a Euclidean V, prove that $|\langle\alpha,\beta\rangle| \le \|\alpha\|\,\|\beta\|$, the equality holds when α, β are lineraly dependent. 3

Group - B

(Vector Calculus - I) (Marks –15)

Answer *any three* questions

7. (a) If $\frac{d\bar{a}}{dt} = \bar{\omega} \times \bar{a}$ and $\frac{d\bar{b}}{dt} = \bar{\omega} \times \bar{b}$, show that

$$\frac{d}{dt}(\bar{a} \times \bar{b}) = \bar{\omega} \times (\bar{a} \times \bar{b}).$$ 2

(b) Show that $\frac{|\dot{\bar{r}} \times \ddot{\bar{r}}|}{|\dot{\bar{r}}|^3}$ is the same at all points of the curve whose vector equation is $\bar{r} = 4\cos t\,\hat{i} + 4\sin t\,\hat{j} + 2t\,\hat{k}$. 3

8. At any instant t, the position vector of a moving particle in a plane, relative to some origin O in the plane is given by $\bar{r} = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ where ω is a constant. Show that –

(a) the velocity $\bar{v}$ of the particle is perpendicular to $\bar{r}$.

(b) the acceleration $\bar{a}$ is directed towards O and has magnitude proportional to the distance of the particle from O.

(c) $\vec{r} \times \vec{v}$ is a constant vector. 2+1+2

9. Prove $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$,

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = \sqrt{x^2 + y^2 + z^2}$. Find $f(r)$ such that $\nabla^2 f(r) = 0$. 3+2

10. Define irrotational and solenoidal vector. Show that $r^n\vec{r}$ is an irrotational vector of any vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and any value of m, but is solenoidal if $n + 3 = 0$. 5

11. (a) If $\vec{F}$ is continuously differentiable vector point function and φ is continuously differentiable scalar point function of (x, y, z), then prove that

$$\vec{\nabla} \times (\varphi \vec{F}) = \vec{\nabla}\varphi \times \vec{F} + \varphi(\vec{\nabla} \times \vec{F}).$$ 3

(b) Show that $\vec{\nabla} \times \{\vec{a}, \vec{r})\vec{a}\} = \vec{0}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{a}$ is a constant vector. 2

PART – II

THIRD PAPER – 2017

(Module - V)

Full Marks - 50

Group-A

(Modern Algebra - II)

(Marks -15)

Answer *any three* questions

1. (a) Let $G = \langle a \rangle$ be a cyclic group of order n. Then prove that $G = \langle a^K \rangle$ if and only if gcd $(k, n) = 1$. 3

(b) Find all generators of Z_{20}. 2

2. (a) Write down the elements of symmetric group S_3 of degree 3. 2

(b) Prove that every proper sub-group of the symmetric group S_3 is cyclic. 3

3. (a) Let H be a subgroup of a group G and $a, b, \in G$, prove that the left cosets aH and bH are indentical if $a^{-1}b \in H$. 3

(b) Given that the set of all 2×2 matrices of the form $\begin{pmatrix} x & y \\ 0 & z \end{pmatrix}$ where x, y, z are integers, form a ring with respect to matrix addition and multiplication. It is a field ? Justify your answer. 2

4. (a) Let R be a ring unity 1. If the product of any pair of non-zero elements of R is non-zero, prove that $ab = 1$ implies $ba = 1, a, b \in R$. 3

(b) Show that every non-zero element of Z_n is a unit or a zero divisor. 2

5. Prove that every field is an integral domain. Is the converse true ? Justify your answer. 3+1+1

Group-B

(Linear Programming and Game Theory)

(Marks – 35)

Answer **any five** questions

6. (a) State the fundamental theorem of L.P.P. 2

(b) Define a convex ployhedron. Prove that any point of a convex polyhedron can be expressed as a convex combination of its extreme points. 2+3

7. (a) Show that the following system of linear equations has a degenerate solution :

$$2x_1 + x_2 - x_3 = 2$$

$$3x_1 + 2x_2 + x_3 = 3.$$ 3

(b) Let $x_1 = 2$, $x_2 = 4$ and $x_3 = 1$ be a feasible solution to the system of equations

$$2x_1 - x_2 + 2x_3 = 2$$

$$x_1 + 4x_2 = 18.$$

Reduce the given feasible solution to a basic feasible solution. 4

8. A soft drink plant has two bottling machines A and B. It produces and sells 500 ml and 800 ml bottles. The following data are available :

Machine	500ml	800ml
A	100/minute	40/minute
B	60/minute	75/minute

The machines can be run 8 hours per day and 5 days per week. Weekly productions of the drink can not exceed 30,00,000 ml and the market can absorb 25,000 bottles of 500 ml and 7,000 bottles of 800 ml per week. Profit on two types of bottles are 15 paise and 25 paise respectively. The planter wishes to maximize his profit to all productions and marketing restrictions. Formulate the above problem as L.P.P. and solve it graphically. 3+4

9. Solve the following L.P.P. by two-phase method : 7

Minimize $z = 3x_1 + 5x_2$

subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \geq 60;$

where $x_1, x_2 \geq 0$.

10. Write the dual of the L.P.P.

Maximize $z = 5x_1 + 2x_2$

subject to $6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \geq 4$; $x_1, x_2 \geq 0$

Solve the dual using simplex method and interpret the solution of the primal. 3+4

11. Identical products are produced in three factories and sent to four warehouses for delivery to the customers. The cost of transportation and capacities are given by the cost matrix as follows :

	W_1	W_2	W_3	W_4	a_i
F_1	3	8	7	4	30
F_2	3	2	9	5	50
F_3	4	3	6	2	80
b_j	20	60	55	40	

(a) Find an optimal schedule of delivery for minimization of cost of transportation.

(b) Find the idle capacity of the warehouses.

(c) Do you anticipate any alternative optimum solution for this problem ? How can the same be identified ; 4+1+1+1

12. A salesman is planning to four cities B, C, D and E from his home city A. The intercity distance are shown in the following table :

City	A	B	C	D	E
A	∞	103	188	136	38
B	103	∞	262	176	52
C	188	262	∞	85	275
D	136	176	85	∞	162
E	38	52	275	162	∞

How should he plan his tour so that –

(a) he visits each of the cities only once ;

(b) travels the minimum distance ? 7.

13. (a) Let $\left[a_{ij}\right]_{m\times n}$ be the pay-off matrix for a two-person zero-sum game, then prove that

$$\min_j . \max_i . a_{ij} \geq \max_i . \min_j . a_{ij}$$ 3

(b) Find the value of the following 2×2 game algebraically by using mixed strategies. 4

$$\begin{array}{cc} & \text{Player B} \\ & \begin{matrix} B_1 & B_2 \end{matrix} \\ \text{Player A} \begin{matrix} A_1 \\ A_2 \end{matrix} & \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \end{array}$$

14. Use the dominance property to simplify the rectangular game with the following pay-off :

Player A

Player B	I	II	III	IV
I	18	4	6	4
II	6	2	13	7
III	11	5	17	3
IV	7	6	12	2

Find its graphical solution. 3+4

THIRD PAPER – 2017

(Module - VI)

Full Marks - 50

R denotes the set of real numbers

Group-A

(Marks -15)

Answer *any three* questions

1. Answer *any one* :

(a) If $\{a_n\}$ is a decreasing sequence of positive numbers, then prove that the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=0}^{\infty} 2^n a_{2^n}$ converge or diverge together. 5

(b) (i) Prove or disprove : If $\{a_n\}$ is a real sequence and if $\lim_{n\to\infty} (n^2 a_n)$ exists in $\mathbb{R}$, then $\sum_{n=1}^{\infty} a_n$ is convergent. 3

(ii) Prove or disprove : There exist no convergent series $\sum_{n=1}^{\infty} a_n$ of real numbers such that $\sum_{n=1}^{\infty} a_{2n}$ is not convergent. 2

2. (a) Test convergence of *any one* of the following : 3

(i) $\sum_{n=1}^{\infty} a_n$,where $a_n = \left\{\frac{1.3.5...(2n-1)}{2.4.6...2n}\right\}^2 \left(\frac{n}{n+1}\right)$

(ii) $\sum_{n=1}^{\infty} a_n$ where $a_n = \begin{cases} 2^{-n-\sqrt{n}} \text{ if n is odd} \\ 2^{-n-\sqrt{n}} \text{ if n is even} \end{cases}$

(b) Show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^n}{(n+1)^{n+1}}$ is a convergent series. 2

3. **(a) Does there exist a function $\varphi:[-1,1]\to\mathbb{R}$ such that** $\varphi'(x) = f(x)\ \forall x \in [-1,1]$ where $f(x) = |x| - [x]\ \forall x \in [-1,1]$? Justify your answer. 2

(b) If f'' is continuous for a function $f : [a, a+h] \to \mathbb{R}$ and $f''(a) \neq 0$, prove that $\lim_{h\to\infty} \theta = \frac{1}{2}$ where θ is given by

$$f(a+h) = f(a) + hf'(a+\theta h),\ 0<\theta<1$$ 3

4. (a) If $\varphi(x)$ is differentiable function for all $x \in \mathbb{R}$ and $a > 0$ is such that $\varphi(0) = \varphi(2a)$, $\varphi(a) = \varphi(3a)$ and $\varphi(0) \neq \varphi(a)$ then show that there is at least one root of the equation $\varphi'(x+a) = \varphi'(x)$ in $(0, 2a)$. 2

(b) For $x > 0$, prove that $0 \le \sin x - x + \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} \le \frac{x^9}{9!}$ 3

Or, Evaluate : $\lim_{x\to 0+} X^{\sin x}$ 3

5. A cone is made from a circular sheet of radius $\sqrt{3}$ by cutting out a sector and keeping the cut edges of the remaining piece together. Find the maximum volume attainable for the cone. 5

Group - B

(Marks - 35)

Answer ***any five*** questions

6. (a) Find the differential equation of the parabolas touching the co-ordinate axes. 4

(b) Solve : $(x^2y^2+xy+1)y\,dx+(x^2y^2-xy+1)x\,dy=0$ 3

7. Reduce the differential equation

$x^2p^2+y(2x+y)p+y^2=0$ to Clairaut's form by substitution $y=u$, $xy=v$ and then solve it $\left(p=\frac{dy}{dx}\right)$.

Obtain also the singular solution, if any. 3+2+2

8. (a) Find the orthogonal trajectories of $r^n\cos n\theta=a^n$ where 'a' is the parameter.

(b) Find the eigenvalues and eigenfunctions for the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\lambda y=0\ (\lambda>0)$ satisfying the boundary conditions $y(1)=0=y(e^{\pi})$. 3+4

9. Using the method of undetermined coefficients solve the equation :

$$\frac{d^2y}{dx^2}+4y=x^2\sin 2x$$ 7

10. Solve :

$$x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log x)$$ 7

11. Apply the method of variation of parameters to solve :

$$x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}$$ 7

12. (a) Solve : $\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+(x^2+1)y=x^3+3x$

by reducing to normal form. 5

(b) Determine whether the equation

$(1+yz)dx+x(z-x)dy-(1+xy)dz=0$

is integrable. 2

13. (a) Solve the simultaneous equations :

$$2\frac{dx}{dt}-2\frac{dy}{dt}-3x=t$$

$$2\frac{dx}{dt}+2\frac{dy}{dt}+3x+8y=2$$ 5

(b) Eliminate arbitrary function f from $z=f(y/x)$ under suitable condition on f. 2

14. (a) Solve :

$(mz-ny)p+(nx-lz)q=ly-mx$, where $p=\frac{\partial z}{\partial x}$, $q\frac{\partial z}{\partial y}$ 3

(b) Find the complete integral of $(p^2+q^2)x=pz$ where $p=\frac{\partial z}{\partial x}, q\frac{\partial z}{\partial y}$. 4

FOURTH PAPER – 2017

(Module - VII)

Full Marks -50

($\mathbb{R}$ denotes the set of real numbers)

Group - A (Marks - 30)

Answer ***any six*** questions

1. (a) Examine whether the points (1, 1), (0, 0) are interior points of the set $\{(x,y)\in\mathbb{R}^2 : |x|\le 1,\ |y|\le 1\}$ in $\mathbb{R}^2$. 2

(b) Let $f : S \to \mathbb{R}$ where $S \subset \mathbb{R}^2$ and (a,b) be an accumulation point of S. If $\lim_{(x,y)\to(a,b)} f(x,y) = L$, then prove that $f(x,\phi(x)) \to L$ as $x \to a$, where ϕ is a real valued function of one variable such that $(x,\phi(x)) \subset S$ for each $x \in$ dom ϕ and $\phi(x) \to b$ as $x \to a$. Here "dom ϕ" denotes domain of ϕ. 3

2. (a) Let $f(x,y) = \begin{cases} x\sin\dfrac{1}{y} + \dfrac{x^2-y^2}{x^2+y^2} & , \text{if } y \neq 0 \\ 0 & , \text{if } y = 0 \end{cases}$

Find the repeated limits, if exist, at the point (0, 0). 2

(b) Show that the function $f(x,y) = |x| + |y|$ is continuous at (0, 0) although the 1st order partial derivatives do not exist at (0, 0). 3

3. State a set off sufficient condition for differentiability of a function of two variables. Are these conditions necessary ? Justify your answer. 2+3

4. Let $f(x,y) = \begin{cases} \dfrac{xy^3}{x^2+y^2} & , \text{if } x^2+y^2 \neq 0 \\ 0 & , \text{if } x^2+y^2 = 0 \end{cases}$

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

Which condition of Young's theorem is not satisfied by f? Justify your answer. 3+2

5. If z is a function of two variables x, y and $x = c\cosh u \cos v$, $y = c \sinh u \sin v$ (c is a real number), show that

$$\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = \frac{c^2}{2}(\cosh 2u - \cos 2v)\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$$

assume that the second order partial derivatives of z are continuous. 5

6. If u is a homogeneous function of x, y, z of degree n having continuous second order partial derivatives and if $u = f(\xi,\eta,\zeta)$ where ξ,η,ζ are the partial derivatives of u with respect to x, y, z respectively, prove that $\xi\dfrac{\partial u}{\partial \xi} + \eta\dfrac{\partial u}{d\eta} + \zeta\dfrac{\partial u}{\partial \zeta} (n \neq 1)$. 5

7. Show that the function $u = x+y+z$, $v = xy+yz+zx$, $w = x^3+y^3+z^3-3xyz$ are not independent. Find the relation between u, v and w. 3+2

8. If $x^2 = vw$, $y^2 = wu$, $z^2 = uv$ and $f(x,y,z) = \phi(u,v,w)$, show that $xf_x + yf_y + zf_z = u\phi_u + v\phi_v + w\phi_w$. 5

9. Sate a set of sufficient condition for the existence and uniqueness of implicit function of two variables. Examine the existence of implict function near the point (0, 1) for the equation $x^2+y^2-1=0$ and also find the implict function and its derivative at (0, 1), if exists. 1+2+1+1

10. Use Taylor's theorem to expand $x^4 + x^2y^2 - y^4$ about (1, 1) up to terms of the degree and find the form of the remainder after two terms. 5

11. Use Lagrange's method to find the shortest distance from the point (0, b) to the parabola $x^2 - 4y = 0$. 5

Group - B
(Marks - 20)

Answer ***any four*** questions

12. (a) Show that the pedal of the curve $r = a(1+\cos\theta)$ with respect to pole is $r = 2a\cos^3\left(\dfrac{\theta}{3}\right)$. 5

13. Find the equation of the curve which has the same asymptotes as those of the curve $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$. 5

14. Find the locus of the centre of curvature of the hyperbola $xy = C^2$. 5

15. If $x^{\frac{2}{3}} + y^{\frac{2}{3}} = c^{\frac{2}{3}}$ is the envelope of the family of the curves $\frac{x^2}{y^2} + \frac{y^2}{b^2} = 1$, prove that the parameters a and b are connected by $a + b = c$ 5

16. Find the intervals in which the curve $y = e^x(\cos x + \sin x)$, $x \in (0, 2\pi)$, is concave upwards or downwards. Find also the points of inflexion, if any. 5

17. Find the centre of gravity of the area enclosed by the curves $y^2 = ax$ and $x^2 = by$. 5

18. Find the area of the smaller portion enclosed by the curves $x^2 + y^2 = 9, y^2 = 8x$. 5

FOURTH PAPER – 2017

(Module - VIII)

Full Marks - 50

Group - A

1. Answer *any one* of the following :

(a) If ds is the distance between two points with spherical co-ordinates which are infinitesimally near each other, find ds in terms of $(dr, d\theta, d\phi)$. 3

(b) Find the equation of the sphere passing through origin and touching $x^2 + y^2 + z^2 = 56$ at the point (2, 4, –6). 3

2. Answer *any two* of the following :

(a) Show that the perpendiculars from the origin to the generators of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ lie upon the cone

$$\frac{a^2(b^2 + c^2)}{x^2} + \frac{b^2(c^2 + a^2)}{y^2} = \frac{c^2(a^2 - b^2)}{z^2}.$$ 6

(b) If a circular cone has three mutually perpendicular generators, find its semi-vertical angle. 6

(c) Show that the plane $z = a$ meets any enveloping cone of the sphere $x^2 + y^2 + z^2 = a^2$ in a conic with focus at $(0, 0, a)$. 6

(d) Prove that the only non-central ruled surface is the hyperbolic paraboloid. 6

Group - B

3. Answer *any one* of the following :

(a) (i) Find the conditions for a system of co-planar forces to be in Astatic equilibrium. 5

(ii) A perfectly rough plane is inclined to horizon at an angle α. Show that the least eccentricity of the ellipse that can rest on the plane is given by $\sqrt{\frac{2\sin\alpha}{1+\sin\alpha}}$. 5

(b) (i) There forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 3$, $2x + y = 1$ and $y - x = 1$. Find the equation of the line of action of the resultant. 6

(ii) Find the positions of equilibrium for a particle resting on a rough plane curve under a given force F. 4

Group – C

4. Answer *any one* of the following :

(a) A ball of unit mass falls from height h upon a fixed horizontal plane with coefficient of restitution 0.25. Show that the whole distance described before it comes rest in $\frac{17}{15}h$. Also, find the whole time taken during the motion. 7

(b) A particle moves in a straight line with an acceleration n^2x towards a fixed origin on the line and is simultaneously acted on by a periodic force $F\cos pt (p \neq n)$ per unit mass. Investigate the motion. State what happens when $p = n$. Cite a physical example in support of your answer. 3+3+1

5. Answer ***any two*** of the following :

(a) (i) A particle falls down a cycloid under its own weight starting from the cusp. Show that on arriving the vertex, the pressure on the curve will be twice the weight of the particle. 4

(ii) Find the accelerations of a particle, along and perpendicular to the radius vector from a fixed origin, moving in a plane curve. 5

(b) (i) A particle is moving with uniform angular velocity ω along the ellipse $\frac{l}{r} = 1 - e\cos\theta$. When the particle is at one end of the latus rectum through the pole, find the component of acceleration towards the pole. 4

(ii) If a particle falls vertically from rest in a medium whose resistance varies as square of the velocity, find the distance fallen through time t. 5

(c) (i) A particle moves on a plane under a force $\vec{F} = \hat{i}X + \hat{j}Y$, where X, Y are such functions of the position (x, y) of the particle at time t, that $Xdx + Ydy$ is an exact differential. Prove that the sum of K.E. and P.E. is constant throughout the motion. 6

(ii) A curve is described by a particle having constant acceleration in a direction inclined at a constant angle to the tangent. Find the curve. 3

(d) (i) The curve $x = a\,(\theta - e\sin\theta)$; $y = a\,(1 - e\cos\theta)$, where a, e are constant and θ is parameter is described by a particle under the action of a force parallel to X-axis. Show that the force will vary as $\left(\dfrac{e - \cos\theta}{\sin^3\theta}\right)$. 5

(ii) A particle moves along X-axis with an acceleration μ / x^3, where $\mu (> 0)$ is constant and x is the distance from origin. If it starts from rest at $x = a$ and the acceleration is towards the origin, find the time taken to reach $x = a/4$ from $x = 3a/4$. 4

C.U. B.Sc. MATHEMATICS HONS. QUESTION PAPERS - 2017

PART – III

FIFTH PAPER – 2017

(Module – IX)

Full Marks – 50

$\mathbb{N}$, $\mathbb{R}$, $\mathbb{Q}$ denote the set of all natural numbers, real numbers and rational numbers respectively

Answers ***Question No. 1*** and ***any four*** from the rest

1. (a) Answer ***any two*** questions :

(i) Correct or justify : Let S be a compact set of real numbers. If each point of S is an isolated point of S then S is a finite set. 3

(ii) Check whether true or false :

There exists no power series $\sum_n a_n x^n$ with radius of convergence 1 which is convergent at both 1 and -1. 3

(iii) Let $f:[a,b]\to\mathbb{R}$ be such a continuous function that for every continuous function $g:[a,b]\to\mathbb{R}$, $\int_a^b f(x)g(x)dx = 0$. Prove that $f(x)=0$ for all $x\in[a,b]$. 3

(iv) Let the function f be uniformly continuous on $\mathbb{R}$ and for each $n\in\mathbb{N}$, $f_n(x) = f\left(x+\frac{1}{n}\right)$ $\forall x\in\mathbb{R}$. Show that $\{f_n\}$ converges uniformly to f on $\mathbb{R}$. 3

(b) Answer ***any two*** questions :

(i) Show that the set $\left\{x\in[-1,1] : \int_{-1}^{x}[t]dt \text{ is continuous at } x\right\}$ is a compact set, where $[t]$ denotes the greatest integer function. 2

(ii) Find the variation function of $f(x)=\begin{cases}2, & 0\le x\le 1\\ 1, & 1<x\le 2\end{cases}$ 2

(iii) Prove or disprove : If $f:[a,b]\to\mathbb{R}$ is Riemann integrable, then f cannot have a discontinuity of 2nd kind. 2

(iv) Show that the series $\sum_{n=1}^{\infty}\frac{\sin nx}{n^2+1}$ has a continuous sum on $\mathbb{R}$. 2

2. (a) Give an example of a compact set of real numbers whose derived set is a countable infinite set. 3

(b) Prove or disprove : The set $\{x\in\mathbb{R}: x^2-2x-15<0\}$ is a compact set. 2

(c) Show that a compact set of real numbers has a greatest element. 2

(d) If A is an infinite compact set or real numbers, prove that every infinite subset of A has a limit point contained in A. 3

3. (a) Correct or justify : let $f:[a,b]\to\mathbb{R}$ be integrable and $g:[a,b]\to\mathbb{R}$ be a bounded function with $f(x)\neq g(x)$ only for $x\in[a,b]\cap\emptyset$. Then $\int_a^b g(x)dx=\int_a^b g(x)dx$. 3

(b) Let $f:[a,b]\to\mathbb{R}$ be a bounded function. Then prove that f is integrable if and only if for any $\in>0$, there exists a partition P of $[a,b]$ such that $U(P,f)-L(P,f)<\in$. 4

(c) Prove or disprove : A continuous function defined on a closed and bounded interval is of bounded variation there. 3

4. (a) Let $f:[0,1]\to\mathbb{R}$ be defined by :

$$f(x)=\frac{1}{2^n},\quad \frac{1}{2^{n+1}}<x\leq\frac{1}{2^n}\text{ for all } n=0,1,2,\ldots\ldots\text{ and } f(0)=1.$$

Show that f is integrable on $[0,1]$. 2

(b) Let f and g be continuous on $[a,b]$ and $\int_a^b f(x)dx=\int_a^b g(x)dx$. Show that there exists c in $[a,b]$ such that $f(c)=g(c)$. 3

(c) Find the value of $\lim_{x\to\infty}\frac{1}{x-3}\int_3^x e^{\sqrt{1+t^2}}dt$. 2

(d) If $f,g:[a,b]\to\mathbb{R}$ are Riemann integrable on $[a,b]$ and g maintains same sign on $[a,b]$, then prove that there exists $\mu\in[m,M]$ such that $\int_a^b f(x)g(x)dx=\mu\int_a^b g(x)dx$

where $m=Inf\{f(x):x\in[a,b]\}$ and

$M=Sup\{f(x):x\in[a,b]\}$. 3

5. (a) If $f:[a,b]\to\mathbb{R}$ is a function of bounded variation then show that f can be expressed as a difference of two monotonic increasing functions on $[a,b]$. 3

(b) Determine the perimeter of the Cardioide

$r=a(1-\cos\theta),\ a>0$. 3

(c) A function f is defined on [0,1] by :

$$f(x)=\begin{cases}\sqrt{1-x^2}, & x\in[0,1]\cap\emptyset\\ 1-x, & x\in[0,1]\setminus\emptyset\end{cases}$$

Show that f is not integrable on [0, 1] 4

6. (a) Let $D\subseteq\mathbb{R}$ and $\{f_n\}$ be a sequence of functions from D to $\mathbb{R}$. Then show that the sequence $\{f_n\}$ is uniformly convergent on D if and only if for any $\varepsilon>0$ there exists a natural number K such that for all $x\in D$.

$$|f_{n+p}(x)-f_n(x)|<\varepsilon \text{ for all } n\geq k,\ p=1,2,3\ldots.. \quad 5$$

(b) State Dini's Theorem on sequence of real valued functions. A sequence of functions $\{f_n\}$ is defined by $f_1(x)=\sqrt{x}, f_{n+1}(x)=\sqrt{xf_n(x)}$

for all $n \geq 1$, $x \in [0,1]$. Use Dini's theorem to prove that the sequence $\{f_n\}$ is uniformly convergent on $[0,1]$. 2+3

7. (a) If $f(x)$ be the sum of the series $\sum_{n=1}^{\infty} ne^{-nx}$, $x > 0$ show that f is continuous on $[\log 2, \log 3]$. Evaluate $\int_{\log 2}^{\log 3} f(x)dx$. 3+2

(b) State Dirichlet's test on uniform convergence for series of functions. Prove that the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n}$ is uniformly convergent on any closed interval $[a,b]$ contained in the open interval $(0,2\pi)$. 2+3

8. (a) (i) Let $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=0}^{\infty} b_n x^n$ be two power series having same radius of convergence $\rho \in (0,\infty)$ and same sum function f in $(-\rho,\rho)$. Then prove that $a_n = b_n$, $\forall n = 0,1,2.....$ 2

(ii) If a power series is neither nowhere convergent nor everywhere convergent, then prove that the radius of convergence remains unaltered on term-by-term differentiation. 3

(b) Starting from the power series expansion of $(1-x^2)^{\frac{-1}{2}}$, $|x| < 1$, derive the power series of $\sin^{-1} x$ together with its range of validity. Hence obtain the sum of the series

$$1+\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{5}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{1}{7}+\cdots\cdots$$ 4+1

FIFTH PAPER – 2017

(Module – X)

Full Marks – 50

Group – A

(Marks – 20)

Section – I

Answer *any one* question

1. (a) Let V and W be vector spaces over a field F. Let $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ be a basis of V and $\beta_1, \beta_2, ..., \beta_n$ be arbitrarily chosen elements (not necessarily distinct) in W. Prove that there exists one and only one linear mapping $T: V \to W$ such that $T(\alpha_i) = \beta_i$ for $i = 1,2,...,n$. 5

(b) A linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3,\ 2x_1 + 3x_2 + x_3,\ 3x_1 + x_2 + 2x_3)$, $(x_1, x_2, x_3) \in \mathbb{R}^3$. Find the matrix of T relative the ordered basis $\{(-1, 1, 1), (1, -1, 1), (1, 1, -1)\}$ of $\mathbb{R}^3$. Hence deduce that T is invertible. 4+1

2. (a) Let V and W be finite dimensional vector spaces over a field F and $\phi: V \to W$ be an isomorphism. Prove that for a set of vectors S in V, S is linearly independent in V if and only if $\phi(S)$ is linearly independent in W. 5

(b) Determine the linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^4$ that maps the basis vectors (0, 1, 1), (1, 0, 1), (1, 1, 0) of $\mathbb{R}^3$ to the vectors (0, 1, 1, 1), (1, 0, 1, 1), (1, 1, 0, 1) respectively, Find KerT and $I_m T$. 3+1+1

Section – II

Answer *any one* question

3. (a) Let H be a subgroup of a group G. Prove that H is normal in G if and only if $h \in H$ and $x \in G \Rightarrow xhx^{-1} \in H$. 5

(b) Let H be a normal subgroup of a group G. Prove that the quotient group G/H is abelian if and only if $xyx^{-1}y^{-1} \in H$ for all $x, y \in G$. 3

(c) Let H be a cyclic subgroup of a group G. If H be normal in G prove that every subgroup of H is normal in G. 2

4. (a) Let G and G′ be two groups and $\phi : G \to G'$ be an onto homomorphism. Let H = Ker ϕ. Prove that $G/H \simeq G'$. 5

(b) (i) Let G be a commutative group of order n. If gcd(m, n) = 1, prove that the mapping $\phi : G \to G$ defined by $\phi(x) = x^m$, $x \in G$ is an isomorphism. 3

(ii) Prove that there does not exist an onto homomorphism from the group $(Z_6, +)$ to the group $(Z_4, +)$. 2

Group – B

(Marks – 15)

Answer *any three* questions

5. (a) If A^i's are component of a contravariant vector and B_{ij} are some functions of the coordinate functions, $x^1, x^2, ..., x^n$ such that $A^i B_{ij}$ are components of a covariant vector, then prove that B_{ij}'s are components of a tensor of type (0, 2). 5

6. If A_i's are component of a covariant vector, show that $\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ are the components of a tensor. 5

7. Calculate the Christoffel symbols [12, 2], $\left\{ \begin{matrix} 2 \\ 1\ 2 \end{matrix} \right\}$ and $\left\{ \begin{matrix} 1 \\ 2\ 2 \end{matrix} \right\}$ in a 3-dimensional Riemannian space in which the line element is given by

$$ds^2 = (dx^1)^2 + (x^1)^2 (dx^2)^2 + (dx^3)^2.$$ 5

8. Prove that $\frac{\partial g^{ij}}{\partial x^k} = -g^{pj}\left\{ \begin{matrix} i \\ p\ k \end{matrix} \right\} - g^{pi}\left\{ \begin{matrix} j \\ p\ k \end{matrix} \right\}$, where the symbols have their usual meaning. 5

9. (a) Prove that the covariant derivative of the metric tensor g_{ij} vanish. 2

(b) If A^i and B^i are two non-null vectors such that $g_{ij}u^i u^j = g_{ij}V^i V^j$, where $u^i = A^i + B^i$ and $V^i = A^i - B^i$, then show that A^i and B^i are orthogonal. 3

Answer either Group – C or Group – D

Group – C

(Marks – 15)

Answer *any one* question

10. (a) If $L\{f(t)\} = F(p)$, then show that $L\left\{\frac{f(t)}{t}\right\} = \int_p^\alpha F(u)du$.

Hence find the Laplace transform of $\frac{e^{-2t} - e^{-5t}}{t}$. 3+2

(b) Using Laplace transform, solve

$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0$, when $y(0) = 3$ and $y'(0) = 0$. 5

(c) Obtain series solution of $\frac{d^2y}{dx^2} - 4(x-1)y = 0$ near the ordinary point x = 1. 5

11. (a) Find inverse Laplace transform of $\frac{2p-5}{4p^2+25} + \frac{4p-18}{9-p^2}$. 5

(b) Using Laplace transform, solve

$\frac{d^2y}{dt^2} + 16y = \cos 3t$, given $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = -2$. 5

(c) Obtain series solution of $\frac{d^2y}{dx^2} - y = x$ near the ordinary point x = 0. 5

Group – D

(Marks – 15)

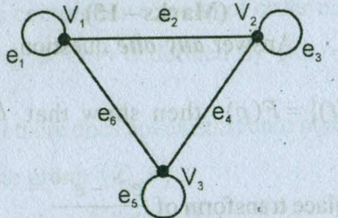
Answer *any three* questions

12. (a) Show that there is no simple graph with six vertices of which the degrees of five vertices are 5, 5, 3, 2 and 1. 2

(b) Show that the number of vertices in a self-complementary graph is either 4K or 4K +1, where K is a positive integer. 3

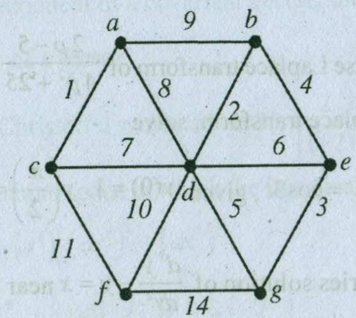
13. (a) Prove that a connected graph with n vertices is a tree if and only if it has (n –1) edges. 4

(b) Find an Euler circuit, if it exists in the following graph. 1



14. If G is a connected plannar graph with $n(\geq 3)$ vertices, e edges, then prove that $e \leq 3n-6$. Also prove that converse of the result is not always true. 3+2

15. Obtain a minimal spanning tree of the following Graph using Kruskal's algorithm : 5



16. Define binary tree. Prove that the number of vertices of a binary tree is odd. Find the number of pendant vertices in a binary tree with 11 vertices. 5

SIXTH PAPER – 2017

(Module – XI)

Full Marks – 50

Group–A

(Vector Calculus – II)

(Marks – 10)

1. Answer ***any two*** questions :

(a) Show that $\iint \vec{F}.\hat{n}\,ds = 64$ where $\vec{F} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the 1st octant between z = 0 and z = 4. 5

(b) State Green's theorem in the xy-plane in vector form. Verify it for the vector function $\vec{F} = (3x^2 - 8y^2)\hat{i} + (4y - 6xy)\hat{j}$ over the region bounded by the curves $y = \sqrt{x}$ and $x = \sqrt{y}$. 5

(c) Verify Stokes' theorem for the vector function $\vec{F} = 3y\hat{i} - xz\hat{j} + yz^2\hat{k}$ where S is the surface of the paraboloid $x^2 + y^2 = 2z$ bounded by z = 2. 5

(d) If $\vec{F} = x\hat{i} - y^2\hat{j} + z^2\hat{k}$, verify Gauss' Divergence Theorem over the region bounded by $x^2 + y^2 = 4,\ z = 0,\ z = 4$. 5

Group – B

(Analytical Statics – II)

(Marks – 20)

*Answer **Question No. 2** and **any two** from the rest*

2. (a) State and prove the Principle of Virtual Work for any system of coplanar forces acting on a rigid body. 6

Or, (b) Find the centre of gravity of the area bounded by the axis of y and the cycloid $x = a(\theta + \sin\theta)$, $y = (1 - \cos\theta)$. 6

3. A parallelogram ABCD is formed of four uniform rods freely jointed at the corners and rests in equilibrium in a vertical plane with AB fixed horizontally. A is attached to the opposite corner C by a light string of length 1 to form the shorter diagonal. If α be the acute

angle of the parallelogram, show that the tension in the string is $\frac{W1\cot\alpha}{2a}$, where W is the total weight of the four rods and AB = a. 7

4. Two equal forces act along the generators of the same system of the hyperboloid $\frac{x^2+y^2}{a^2}+\frac{z^2}{b^2}=1$ and the plane z = 0 at the extremities of the perpendicular diameters of the circle $x^2+y^2=a^2$. Show that the pitch of the equivalent wrench is $\frac{a^2b}{a^2+2b^2}$. 7

5. A heavy uniform rod AB can move freely in a vertical plane about a hinge at A and has a string attached to the end B of the rod. The string after passing over a small smooth pulley at a point C, vertically above A, is attached to a weight. Show that the inclined position of equilibrium of the rod is unstable. 7

6. Three forces each equal to Q, act on a body, one at the point (1,1,0) parallel to Oy, the second at the point (0,1,0) parallel to Oz, the third at the point (0,0,1) parallel to Ox, the axes being rectangular, find the equation of the central axis. 7

Group – C

(Analytical Dynamics of a Particle – II)

(Marks – 20)

Answer ***Question No. 7*** and ***any two*** from the rest

7. (a) If a particle moves in a plane under a radial acceleration F towards a fixed point O of that plane and a cross-radial acceleration T in the increasing direction of the vectorial angle of the position of the particle, show that differential equation of path is given by $\frac{d^2u}{d\theta^2}+u=\frac{F-\frac{T}{u}\frac{du}{d\theta}}{h^2u^2}$, where symbols have usual meanings. 4

Or, (b) Show that the time of description of an arc of a parabolic orbit by a planet moving under the inverse square law is $\frac{\sqrt{2a^3}}{\mu}\left(\tan\frac{\theta}{2}+\frac{1}{3}\tan^3\frac{\theta}{2}\right)$, symbols having their usual meanings. 4

8. Define phase space. What do you mean by equilibrium points of a Dynamical System? Sketch the phase portrait and classify the equilibrium point for the given dynamical system.

$$\frac{dx}{dt}=x+4y,\quad \frac{dy}{dt}=4x+y$$ 8

9. A spherical raindrop, falling freely, receives in each instant an increase in volume equal to λ times its surface at that instant. Find the velocity at time t, and the distance fallen through in that time. 8

10. A particle is projected with a velocity whose horizontal and vertical components are u and v in a medium whose resistance per unit mass is k-times its speed. Find Cartesian equation of path of the particle. Show also that if k is small, the horizontal range is $\frac{2uv}{g}-\frac{8uv^2k}{3g^2}$ approximately.

4+4

11. A particle of mass m moving under a central force $m\mu u^4\left(1-\frac{10}{9}au\right)$ is projected from an apse at a distance 5a from the centre of force with the velocity which is equal to $\sqrt{\frac{5}{7}}$ of the velocity in a circle at the same distance, show that the polar equation of the orbit is $r=a(3+2\cos\theta)$. 8

SIXTH PAPER – 2017

(Module – XII)

Full Marks – 50

Group – A

(Hydrostatics)

Marks – 25

Answer ***any two*** questions, taking ***one*** from each **Section**

Section – I

1. (a) An elliptic lamina is completely immersed vertically in a liquid with its minor axis horizontal and its centre is at a depth h below the effective surface. Determine the position of the centre of pressure. 7

(b) A hollow cone, vertex upwards, is three-quarters full of water and is set rotating about its axis which is vertical, with an angular velocity $\sqrt{\frac{8g}{3h}}\cot\alpha$, where α is the semi-vertical angle and h is the height of the cone. Shw that the ratio of the thrust on the base to the weight of the water in the vessel is 10 : 3. 8

2. (a) For a body floating freely in a homogeneous fluid at rest under gravity, prove that $HM = \frac{AK^2}{V}$, where the symbols have their usual meaning. 8

(b) A hollow sphere of radius *a*, half filled with liquid is made to rotate with angular velocity ω about its vertical diameter. If the lowest point of the sphere is just exposed, show that $2g = a\omega^2\left(2-\sqrt[3]{4}\right)$ 7

Section – II

3. (a) Prove that the surface of separation of two liquids of different densities, which do not mix, at rest under gravity is a horizontal plane. 5

(b) A vertical circular cylinder of height 2h and radius r, closed at the top, is just filled by equal volumes of two liquids of densities ρ and σ. Show that if the axis be gradually inclined to the vertical, the pressure at the lowest point of the base will never exceed $g(\rho+\sigma)\left(r^2+h^2\right)^{1/2}$ 5

4. (a) If, near the earth's surface, gravity be assumed to be constant, and the temperature in the atmosphere to be given by $t = t_0\left(t - \frac{z}{nH}\right)$, where H is the height of the homogeneous atmosphere, show that the pressure in the atmosphere will be given by the equation $p = p_0\left(1-\frac{z}{nH}\right)^n$. 5

(b) Prove that, if the temperature in the atmosphere falls uniformly with the height ascended, the height of a station above sea level is given

by $z = a\left\{1-\left(\frac{h}{h_0}\right)^m\right\}$, where h and h_0 are the readings of the barometer at the station and at sea level respectively, and a, m are constants. 5

Group – B

(Rigid Dynamics)

Marks – 25

Answer *any two* questions, taking *one* from each **Section**

Section – I

5. (a) State D'Alembert's Principle. Deduce the general equations of motion from D'Alembert's Principle. 2+5

(b) A solid homogeneous cone of height 'h' and vertical angle 2α oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}\left(4+\tan^2\alpha\right)$. 8

6. (a) What is meant by principle axes of a given material system at a point? Find the condition so that a given straight line may be a principal axis of the material system at any point of its length and if so find the other two principal axes. 2+6

(b) A plank of mass M is initially at rest along a straight line of greatest slope of a smooth plane inclined at an angle 'α' to the horizon and a man of mass M′, starting from the upper end walks down the plank so that it does not move. Show that he will reach the other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$ 7

Section – II

7. (a) A rough uniform rod of length 2a is placed on a rough table at right angles to its edge; if its centre of gravity be initially at a distance b beyond the edge, show that the rod will begin to slide when it has turned through an angle $\tan^{-1}\frac{\mu a^2}{a^2+9b^2}$, where μ is the coefficient of friction. 5

(b) If a sphere be projected up an inclined plane, for which $\mu = \frac{1}{7}\tan\alpha$, with velocity V and an initial angular velocity ω (in the direction in which it would roll up), and if $V > a\omega$, show that the friction acts downwards at first, and upwards afterwards and prove that the whole time during which the sphere rises is $\frac{17V + 4a\omega}{18g\sin\alpha}$. 5

8. (a) A uniform disc of radius a is rolling without slipping along a smooth horizontal table with velocity V, when the highest point becomes suddenly fixed. Prove that the disc will make a complete revolution round the point if $V^2 > 24ag$ where g is the acceleration due to gravity. 5

(b) Two equal rods AB and AC are freely jointed at A and are placed on a smooth table so as to be at right angles. The rod AC is struck by blow at C in a direction perpendicular to itself. Show that the resulting velocities of the middle points of AB and AC are in the ratio 2 : 7. 5

SEVENTH PAPER – 2017

(Module – XIII)

Full Marks – 50

$\mathbb{N}, \mathbb{Z}, \mathbb{R}, \mathbb{C}$ respectively denote the set of natural numbers, integers, real numbers and complex numbrs.

Group – A

(Analysis – IV)

(Marks – 20)

Answer *any two* questions

1. (a) Let $f:[a,b] \to \mathbb{R}$ be a function such that $\lim_{x\to b^-} f(x) = \infty$ and f is Riemann integrable over $[a, b-\varepsilon]$ for all $\varepsilon \in (0, b-a)$. If $\lim_{x\to b^-}(b-x)^p f(x)$ exists finitely and $p < 1$, show that $\int_a^b f(x)dx$ is absolutely convergent. 3

(b) Examine the convergence of the integral $\int_0^\infty \frac{\sin x}{\sqrt{x + \cos x}}dx$. 3

(c) (i) Show that $\int_3^\infty \frac{x^2}{\sqrt{x^7 + 1}}dx$ is convergent. 2

(ii) Test the convergence of the integral $\int_0^1 \frac{dx}{\sqrt{e^x - \cos x}}$ 2

2. (a) Prove that $\int_0^\infty e^{-x}x^{n-1}dx$ is convergent if and only if $n > 0$. 4

(b) Use Dirichlet's test to show that $\int_1^\infty \frac{\sin(x + x^2)}{x^p}dx$ converges if $p > -1$. 3

(c) If f is continuous on [0, 1], show that $\int_0^1 \frac{f(x)dx}{\sqrt{1 - x^2}}$ is convergent. 3

3. (a) Obtain the Fourier series expansion assuming f(x) is periodic of period 2π on $\mathbb{R}$ of the function $f(x) = x\sin x$ on $[-\pi, \pi]$.

Hence deduce that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots$ 4+3

(b) Show that $\iint_E \frac{dx\,dy}{(1+e^y)\sqrt{1 - x^2 - y^2}} = \frac{\pi}{2}\log\frac{2e}{1+e}$ where E is the region given by $x \ge 0$, $y \ge 0$ and $x^2 + y^2 \le 1$. 3

4. (a) Show that the volume of the solid bounded by the cylinder $x^2 + y^2 = 2ax$ and the paraboloid $y^2 + z^2 = 4ax$ is $(3\pi + 8)\frac{2a^3}{3}$. 5

Or, Compute $\iiint_E \frac{dx\,dy\,dz}{x^2+y^2+(z-2)^2}$ where E is the sphere $x^2+y^2+z^2 \le 1$. 5

(b) Show that the area of the part of the surface of the cone $x^2 = y^2+z^2$ inside the cylinder $x^2+y^2=a^2$ is $2\pi a^2$. 5

Or, Find the area of the part of the surface of the paraboloid $4ax = y^2+z^2$ enclosed by the cylinder $y^2 = ax$ and the plane $x = 3a$. 5

Group – B

(Metric Space)

(Marks – 15)

5. Answer *any three* questions :

(a) (i) Let X be the set of all real sequence $\{x_n\}_n$ with $|x_n| \le 1$, for all n. Examine whether the mapping defined by $d(x,y) = \sum_{i=1}^{\infty} \frac{1}{2^n}|x_n - y_n|$, for all $x = \{x_n\}_n, y = \{y_n\}_n \in X$ is a metric on X. 3

(ii) Let $A = \{(x,y) : x^2+y^2=1\}$ and $B = \{(x,y) : (x-1)^2+y^2=1\}$. Find the diameter of the sets $A \cup B$ and $A \cap B$. 2

(b) (i) Let α be a point in a metric space (X,d) and $0<p<q$. Prove that the set $\{x \in X : p < d(x,\alpha) < q\}$ is an open set in (X, d). 3

(ii) Prove or Disprove : Boundaries of two disjoint sets in a metric space cannot be equal. 2

(c) (i) Let (X, d) be a metric space and $A \subseteq X$. Then prove that $d(x,A) = d(x,\bar{A})$ for $x \in X$ where $\bar{A}$ is the closure of A. 3

(ii) Let $\{x_n\}_n, \{y_n\}_n$ be two sequences in a metric space (X, d) such that $d(x_n, y_n) \to 0$ as $n \to \infty$. If $\{x_n\}_n$ is a Cauchy Sequence, then prove that $\{y_n\}_n$ is also a Cauchy Sequence.

(d) (i) In the Metric Space ($\mathbb{R}$, d) with usual metric d, consider the sequence $\{F_n\}_n$ of sets where

$$F_n = \left[-3-\frac{1}{n}, -3\right] \cup \left[3, 3+\frac{1}{n}\right], \forall\ n \in \mathbb{N}.$$

Show that $\bigcap_{n=1}^{\infty} F_n$ is not singleton. Give reason. 3

(ii) Show that every finite set in a metric space is closed. 2

(e) Let $\mathbb{N}$ be the set of natural numbers and d be the metric defined on $\mathbb{N}$ by $d(m,n) = \frac{1}{m} - \frac{1}{n}$, $\forall\ m,n \in \mathbb{N}$. Prove that ($\mathbb{N}$, d) is an incomplete metric space. 5

Group – C

(Complex Analysis)

(Marks – 15)

6. Answer *any three* questions :

(a) (i) For the point $\frac{\sqrt{3}-i}{2}$ in the complex plane $\mathbb{C}$, find the corresponding image point on the Riemann Sphere

$$x^2+y^2+\left(z-\frac{1}{2}\right)^2 = \frac{1}{4}.$$ 2

(ii) Using definition show that the function defined by $f(z) = z + i(z+\bar{z})$ $(z \in \mathbb{C})$ is continuous at $z = 1+i$. 3

(b) Let $f(z) = u + iv \begin{cases} \frac{x^2y^5(x+iy)}{x^4+y^{10}}, z \ne 0 \\ 0, \quad x = 0 \end{cases}$, $z = (x,y) \in \mathbb{C}$. Show that real valued functions u and v satisfy Cauchy-Riemann equations at the origin but $f'(0)$ does not exist. 3+2

(c) Show that function $f : \mathbb{C} \to \mathbb{C}$ such that $f(z) = u(x,y) + iv(x,y)$ is continuous at $z_0 = x_0 + iy_0$ if and only if u and v are both continuous at (x_0, y_0). 5

(d) (i) Let $f : G \to \mathbb{C}$, where $f(x+iy) = u(x,y) + iv(x,y)$ be a function of complex variable defined on a region G in $\mathbb{C}$ such that 'f' is differentiable at $z = (\alpha, \beta) \in G$. Prove that u(x, y) are differentiable at (α, β). 3

(ii) If f(z) is analytic in a domain D, then show that f(z) must be constant in D if arg f(z) is constant. 2

(e) If $v(x, y) = e^{-2xy} \sin(x^2 + y^2)$, Show that v satisfies Laplace's equation and find an analytic function of which v is the imaginary part. 2+3

SEVENTH PAPER – 2017

(Module – XIV)

Full Marks – 50

Group – A

(Probability)

Marks – 50

Answer *any two* questions

1. (a) State and prove Baye's Theorem based on conditional probability. 2+4

(b) An even no. of cards are drawn from a full pack of cards. What is the probability that half of them will be black and half of them will be red? 4

(c) Two persons A and B have agreed to meet at a definite spot between 12 and one o'clock. It has been mutually agreed that the first one to come will wait for 20 minutes and then leave the place. What is the probability of a meeting between A and B if the arrival of each during the indicated hours can occur at random and also the times of arrival are independent? 5

2. (a) Show that the most probable number of successes in a Bernoullian sequence of *n* trials in the integers is determined by the inequality $(n+1)p - 1 \le i_m \le (n+1)p$ 5

(b) Calculate mean and variance of normal distribution using moment generating function. 2+3

(c) A random variable *X* is uniformly distributed over the interval

(0,2). Find the distribution function of the larger root of the quadratic equation $t^2 + 2t - X = 0$. 5

3. (a) Show that the first absolute moment about the mean for normal (μ, σ) distribution is $\sqrt{\frac{2}{\pi}}\sigma$. 4

(b) if (X, Y) be a two dimensional random variate, show that

$$[E(XY)]^2 \le E(X^2).E(Y^3)$$

Hence prove that $-1 \le \rho \le 1$, where 'ρ' is the correlation coefficient between *X* and *Y*. 4+2

(c) If *X* is a $\Gamma(n)$ variate, prove that $P(0 < X < 2n) \ge \frac{n-1}{n}$. 5

4. (a) If *X* and *Y* are independent variate both uniformly distributed over (0, 1), find the distribution of $X + Y$ and $X - Y$. 6

(b) Calculate median and mode of the distributio having given p.d.f.

$$f(x) = \lambda e^{-\lambda x}; (\lambda > 0), x > 0.$$ 2+2

(c) State Chebyshev's inequality and its link with weak law of large number. Use this to show that if a die is thrown 3600 times, then the probability that the number of sixes lies between 550 and 650 is at least $4/5$. 1+2+2

Group – B

(Statistics)

Marks – 20

Answer *any one* question

5. (a) Explain the following terms (*any two*): 3×2

(i) Sample statistic, population parameter.

(ii) Sampling distribution

(iii) Method of maximum likelihood

(iv) Law of statistical regularity.

(b) Prove that sample mean $(\bar{x})$ is an unbiased estimator for the population mean whereas sample variance is not. 2+3

(c) A population is defined by the density function

$$f(x,\alpha)=\frac{x^{l-1}e^{-x/\alpha}}{\alpha^{l}\Gamma(l)}\quad 0<x<\infty$$

l being a known constant. Estimate the parameter α by the method of maximum likelihood and show that the estimate is consistent and unbiased. 5

(d) The variable X is normally distributed with mean 68cm and s.d. 2.5cm. What should be the size of the sample whose mean shall not differ from the population mean by more than 1cm with probability 0.95? 4

6. (a) State Neyman Pearson lemma to obtain the best powerful critical region for testing hypothesis. Construct a test for the null hypothesis $H_0 : m = m_0$ against alternative $H_1 : m \neq m_0$ for a normal (m,σ) population by the method of likelihood ratio testing. 2+6

(b) Given the probability density function $f(x,\theta)=\theta e^{-\theta x}$, $0 \le x < \infty, \theta > 0$ of a population. The null hypothesis $H_0 : \theta = 2$ against the one side alternative hypothesis $H_1 : \theta > 2$ will be tested on the following procedure :

H_0 would be rejected if a sample drawn from the population (X) is greater than or equal to 6. Find the type 1 error and power of test. 5+2

(c) What do you mean by Interval Estimation? Find an approximate confidence interval with a given confidence coefficient $1-\in (0<\in 1)$ for 'p' of a binomial (n,p) population assuming n is known and large. 5

EIGHTH PAPER – 2017

(Module – XV)

Full Marks – 50

Group – A

(Marks – 25)

Answer *any five* questions

1. (a) Prove that $E \equiv 1+\frac{\delta^2}{2}+\delta\sqrt{1+\delta^2/4}$ 3

(b) Prove that $\mu^2 \equiv 1+\delta^2/4$ 2

(Symbols have their usual meanings)

2. Define divided difference. Derive divided difference formula for three arguments. 2+3

3. Establish Lagrange's polynomial interpolation formula (without error term). Is this polynomial unique? 4+1

4. Establish the iteration formula for Newton-Raphson method. Write down the condition of convergence. What is the geometrical interpolation of Newton-Raphson method? 3+1+1

5. Describe the power method to calculate the numerically greatest eigenvalue of a real non-singular square matrix of order n. How do you find its numerically least eigenvalue? 4+1

6. What is degree of precision of a mechanical quadrature formula? Stating the error term of Simpson's $\frac{1}{3}$-rd rule, comment upon its degree of precision. 3+2

7. You are given a system of linear equations of the form :

$$\sum_{j=1}^{n} a_{ij}x_j = a_i, n+1,\ i=1,2,3,\ldots\ldots\ldots,n$$

If n be large, discuss the methods adopted for solving such equations in preference to the usual matrix inversion. 2

(b) Discuss the convergence of Gauss-Seidel iteration process. 3

8. Solve by Euler's method the following differential equation :

$$\frac{dy}{dx} = xy, \ y(0) = 1, \text{ for } x = 1 \text{ with } h = 0.2.$$

Give the result correct to four decimal places. 5

9. (a) Use Picard's method to compute y(0.1) from the IVP

$$\frac{dy}{dx} = 1 + xy, \text{ given } y = 1 \text{ when } x = 0$$

correct up to 5 significant figures. 4

(b) Does a single-step method for solving an IVP always converge? Justify your answer. 1

Group – B

(Marks – 25)

Section – I

Answer *any two* questions

10. (a) Multiply : $(11011.101)_2$ by $(101.111)_2$. 2

(b) Evaluate : $(162)_8 + (537)_8$ 2

(c) What is a 'computer word'? Write the full form of "http". 1+1

(d) What will the output of the following FORTRAN program? 3

```
C     PROGRAM OUTPUT
      INTEGER X, Y, Z
      X=50
      Y=10
      X=X+Y
      Y=X+Y
      Z=X+Y
      PRINT *, 'X=', X, 'Y=', Y, 'Z=', Z
      STOP
      END
```


Or, Write the output of the following C program

```
# include <stdio.h>
main( )
{
int a, b, c ;
a = 12 ;
++ a ;
b = a ++ ;
c = ++ a ;
printf ("a = %d, b = %d, c = %d", a, b, c) ;
}
```

3

11. (a) Write an algorithm to compute combination of n things taken r at a time $(n \geq r)$. 5

(b) Write an efficient program in FORTRAN 77/90 or C to generate the successive terms of the series

$2+4+2+4+2+4+$ ....

and find the sum of N terms of the series. 4

12. (a) Draw a flow chart to arrange the following list of numbers in ascending order :

81, 24, 38, 55, 3, 103, 18, 79, 10, 68 4

(b) Write an efficient FORTRAN 77/90 or C program to solve numerically the following differential equation by 4th order Runge-Kutta method correct up to 6D :

$$\frac{dy}{dx} = \frac{x+y}{x^2+y^2}$$

for x = 0.1 (0.1)1.0 given that $y = 1.0$ for $x = 0.0$. 5

13. (a) Write a suitable program in C or in FORTRAN to find the sum of the infinite series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\cdots$ correct to six decimal places for $x = 0.25$. 4

(b) Draw a flow chart to find the real roots of the equation $ax^2 + bx + c = 0$ for given real values of a, b and c. 5

Section – II

Answer ***any one*** question

14. (a) Stating Huntington's postulates for Boolean Algebra B, deduce the following :

$$a + a.b = a \text{ for all } a, b \in B \qquad 2+2$$

(b) Express $E = [x'.y + (x.z)'].(x + y.z)'$ as a full Disjunctive Normal Form (DNF) involving three variables x, y and z. 3

15. (a) In Boolean Algebra $B(+, \cdot, ')$, prove that $a + (b + c) = (a + b) + c$ for all $a, b, c \in B$. 4

(b) Find a simplified circuit using two switches which is equivalent to the following switching circuit. 3