2023

PHYSICS — HONOURS

Paper : DSE-B-1.1 and DSE-B-1.2

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Paper : DSE-B-1.1

(Astronomy and Astrophysics)

Full Marks : 65

Answer ***question nos.*** **1** and **2**, and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Given that the Sun has a luminosity of 3.9×10^{26}W and an absolute magnitude of 5. Find the absolute magnitude and luminosity of a star at a distance of 4pc and apparent magnitude of 2.

(b) What should be the radius of the Sun if the escape velocity from the surface were to exceed the speed of light? (Take $G = 6.67 \times 10^{-11}$ m^{3}s^{-2} kg^{-1} and solar mass $= 1.99 \times 10^{30}$ kg).

(c) Write down the virial theorem for a gravitationally bound system. Explain each term.

(d) What is the physical significance of Jeans Mass?

(e) Explain why the solar system doesn't expand even if the whole universe is expanding.

(f) Assume a value of 70 km/s/Mpc for the Hubble's constant and find the time required in a $k = 0$, $\Omega_\Lambda = 1$ universe for the galaxies in the nearby Virgo Cluster (at 15 Mpc from us) to reach fadeout at the event horizon.

(g) According to the present estimates the value of Hubble's constant is 60 km s^{-1} Mpc^{-1} $< H_0 <$ 80 km s^{-1} Mpc^{-1}. Calculate the approximate age of the Universe from it.

2. Answer ***any three*** questions :

(a) Draw the axes of the Hertzsprung-Russell (H-R) diagram with appropriate labels and show the locus of a star on that diagram during its hydrogen burning lifetime. Also show the approximate positions of white dwarfs and the supergiants in your sketch. 5

(b) Using thermodynamic argument, show that the density $\rho(t)$ of an expanding universe will satisfy

$$\dot{\rho} + 3\frac{\dot{R}}{R}\left(\rho + P/c^2\right) = 0,$$

where P is the pressure, R is the scale factor and dots denote differentiation with respect to cosmic time. You can assume that, as the universe expands the energy of a comoving volume is conserved. 5

(c) Write a short note about Cosmic Microwave Background (CMB) mentioning its origin, discovery and significance in understanding current model of cosmology. 5

(d) Obtain an expression for the free fall time T_{FF} of a spherical mass of gas which collapses without any outward support of pressure. 5

(e) Consider a spherical galaxy whose gravitational potential is given by $\phi(r) = \dfrac{-GM}{\sqrt{r^2 + a^2}}$, where '$a$' is a constant. Compute the mass of the galaxy interior to a radius r. Demonstrate that M is the total mass of the galaxy, i.e., at r tending to infinity. Compute the total potential energy of the galaxy. 3+2

3. (a) Consider a spherical cloud having a constant density ρ and a constant temperature T. Assume the cloud to have a mass M and radius R. Write down the expressions for the gravitational potential energy E_G and the thermal kinetic energy E_T of the cloud.

(b) According to virial theorem, the cloud will be stable if $2E_T = -E_G$. Hence, the condition for collapse would be $-E_G > 2E_T$. Use this condition to obtain the expression for Jeans mass.

(c) In the interstellar medium, we typically have $\rho \sim 10^{-24}$ g/cc and $T \sim 100$ K. Estimate the value of the Jeans mass. Take $\mu = 1$. 4+3+3

4. (a) What are the boundary conditions required to solve the differential equations of stellar structure?

(b) Obtain the equation of state of a main sequence star assuming that it is composed of classical, non-relativistic, ideal gas of completely ionized hydrogen, helium and heavy elements.

(c) Using the stellar hydrostatic equilibrium equation, calculate the expression for gas pressure at halfway between the centre and the surface of the Sun. Assume Sun's density to be uniform throughout. 3+3+4

5. Consider a newly formed globular star cluster, with a total mass 10^6 M_{sun}. Assume that stars form with a range of masses, between 0.1 and 100 M_{sun}, (the lower limit comes from the temperature requirement for thermonuclear reactions, and the upper limit comes from radiation pressure limit). The relative number distribution of stars as a function of mass is described by the initial mass function. There is some evidence that this function is universal, i.e., that it has the same form at all locations, and perhaps at all times in the past (though with some differences). Observations suggest that the number of stars with mass between m and $m + dm$ is given by $dN/dm = Am^{-2.35}$, where A is a constant. This is called the Salpeter initial mass function (IMF). Thus low-mass (red) stars are much more common than high mass (blue) stars.

(a) Find the constant A.

(b) What fraction of stars have mass above 8 M_{sun} (the lower limit for core collapse supernova)?

(c) Find the total luminosity of the cluster, assuming that all its stars are on the main sequence, and a mass luminosity relation $L \sim m^4$.

(d) What fraction of the luminosity is contributed by stars more massive than 5 M_{sun}? 3+2+3+2

6. A spherical source of radiation (with radius R) has a uniform intensity I_ν.

(a) Show that the total flux of radiation from the source at a distance $r > R$ from the centre of the source will be

$$F_\nu(r) = \pi I_\nu \left(\frac{R}{r}\right)^2 .$$

(b) What is the energy density of radiation $u_\nu(r)$ at a distance r expressed in terms of I_ν?

(c) What is the pressure $P_\nu(r)$ in terms of I_ν?

(d) How is P_ν related to u_ν at $r = R$? What happens to the relation when $r >> R$? 4+2+2+2

7. Assume that the density in a star varies linearly with radius as

$$\rho(r) = \rho_c\left(1 - \frac{r}{R}\right),$$

where ρ_c is the central density and R is the radius of the star.

(a) How is the mass M of the star related to its radius R?

(b) Show that the pressure is given by

$$P(r) = \frac{5\pi G\rho_c^2 R^2}{36}\left(1 - \frac{24}{5}\frac{r^2}{R^2} + \frac{28}{5}\frac{r^3}{R^3} - \frac{9}{5}\frac{r^4}{R^4}\right)$$

(c) Show that the central pressure P_c can be written as

$$P_c = \frac{5}{4\pi}\frac{GM^2}{R^4}.$$ 3+5+2

8. Given that, the time evolution of the scale factor $R(t)$ of an expanding universe satisfies the equation

$$\frac{\dot{R}^2}{R^2} - \frac{8\pi G}{3}\rho_m = -K\frac{c^2}{R^2},$$

where K is a constant that specifies the geometry of the universe, c is the speed of light and ρ_m is the mass density given by the conservation of mass in a comoving volume as

$$\rho_m(t)R^3(t) = \rho_{m0}R_0^3,$$

where we have denoted $\rho_m(t_0)$ and $R(t_0)$ by ρ_{m0} and R_0. Here t_0 is the present time and note that at the beginning i.e., at $t = 0$, $R = 0$ (the big-bang condition).

(a) Show that, K can be expressed in terms of the present conditions as

$$K = \frac{H_0^2 R_0^2}{c^2}(\Omega_0 - 1),$$

where $\Omega_0 = 8\pi G\rho_{m0}/3H_0^2$ is the ratio of the actual mass density to the critical value required for closure and H_0 is the value of present-day Hubble parameter.

(b) Introduce the dimensionless variables $\chi = R/R_0$ and $\tau = H_0 t$ and show that the differential equation for R can be transformed into the form

$$\left(\frac{d\chi}{d\tau}\right)^2 - \frac{\Omega_0}{\chi} = 1 - \Omega_0 .$$

(c) Take a special case of $\Omega_0 = 1$ and solve the above differential equation to show that the present age of the universe in this case is

$$t_0 = \frac{2}{3} H_0^{-1} .$$

3+4+3

Paper : DSE-B-1.2

(Nuclear and Particle Physics)

Full Marks : 65

Answer ***question nos.*** **1** and **2**, and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) What do you mean by mirror nuclei? What information can be obtained from their mass difference?

(b) Calculate the threshold energy for the nuclear reaction $^{14}N(n, \alpha)\ ^{11}B$ in MeV. The masses of different nuclei in a.m.u are as follows :

$M(^{14}N) = 14.007550$, $M(n) = 1.008987$, $M(\alpha) = 4.003879$ and $M(^{11}B) = 11.012811$.

(c) Using single particle shell model, calculate the spin and parity of $^{19}_{9}F$ nucleus in its ground state.

(d) Calculate the energy of emergent proton from a cyclotron. Given magnetic field is 0.72 T and extreme radius is 1 m. [Mass of proton = 1.673×10^{-27} kg].

(e) Explain why a Geiger-Muller counter cannot measure the energy of moving charge particle.

(f) Explain briefly why free quarks are not observed in nature.

(g) The decay $\Xi^- \to \Lambda^\circ + \pi^-$ is observed in nature, whereas the apparently similar decay $\Xi^- \to \eta^\circ + \pi^-$ is never observed. Why?

2. Answer ***any three*** questions :

(a) A flux of 10^{12} neutrons / m^2 emerges each second from a port in a nuclear reactor. If these neutrons have a Maxwell-Boltzman energy distribution corresponding to $T = 300$ K;

(i) calculate the average velocity of a neutron.

(ii) calculate the density of neutrons in the beam. 2+3

(b) (i) Why pair production of γ-ray cannot occur in vacuum?

(ii) A γ-radiating source (say ^{60}Co) is lost in a laboratory. The available detectors at hand are a Ge-based solid state detector and a Bi-based scintillation detector. Assuming both are equally easy to handle, which one will you choose to find the lost radioactive source? Justify your answer. 3+2

(c) (i) Write down the differences between compound and direct nuclear reactions.

(ii) What do you mean by thermal neutrons? Indicate their key role in nuclear reaction. 2+(1+2)

(d) What is meant by dead time in Geiger-Muller counter? A Geiger-Muller counter has a dead time 400 μs. What are the true counting rates when the observed rates are 100 per minute? 2+3

(e) Check whether the following reactions are allowed or forbidden. Justify your answer.

(i) $\pi^+ + n \to \Lambda^o + K^+$

(ii) $K^- + p \to K^+ + \Xi^-$

(iii) $n \to p + e^-$. 2+2+1

3. (a) Why does the binding energy per nucleon for medium-sized nuclei remain relatively constant? How do you explain the fall of the binding energy curve for lighter as well as heavier nuclei? Explain from the binding energy curve why energy is released in fission and fusion?

(b) 'While ^{64}Cu can decay either by β^+ or β^-, the ^{73}Ga decays by β^- only.'— Explain. (2+3+2)+3

4. (a) Derive an expression for the threshold energy of an endoergic reaction. Show that the kinetic energy of the outgoing particle will be a double-valued function of the kinetic energy of the incoming particle within a limited range.

(b) The ${}^{10}_{5}\mathrm{B}(\alpha, p)\,{}^{13}_{6}\mathrm{C}$ reaction shows among others a resonance for an excitation energy of the compound nucleus of 13.23 MeV. The width of this level as found experimentally is 130 KeV. Calculate the mean life of the nucleus for this excitation.

(c) Explain briefly why the capture cross-section of a nucleus for thermal neutrons is often very large. (3+2)+3+2

5. (a) What are the primary processes by which a heavy charged particle loses its energy while passing through matter? Draw a typical curve showing the variation of stopping power as a function of penetration depth.

(b) Write down the processes by which γ-ray interacts with matter. Draw the relative cross-sections of these processes as a function of γ-ray energy.

(c) A beam of γ-radiation having photon of energy 510 KeV is incident on a foil of Aluminium. Calculate the wavelength of the scattered radiation in a direction making 90° with the incident beam. What is the kinetic energy of the re-coiling electron? (2+1)+(2+2)+(1+2)

6. (a) Draw the characteristic curve of a Geiger-Muller counter.

(b) An organic quenched GM-counter operates at 1 kV and has a wire of diameter 0.2 mm. The radius of the cathode is 20 mm and the counter has a lifetime of 10^9 counts. What is the maximum radial field? How long will the counter last, if used for 30 hours / week, on an average, at 3000 counts per minute?

(c) Give an account of the working principle of scintillation counter. How can γ-ray energies be determined with its help? 2+(2+2)+(2+2)

7. (a) Explain the fundamental principle behind the operation of a betatron. In what respect does it differ from a synchrotron? State its important applications.

(b) A betatron has a magnetic current supply frequency of 60 Hz and the peak magnetic flux density at the orbit is 0.5 Wb/m^2. If the radius of the electron orbit is 0.75 m, calculate (i) the final energy of the electrons and (ii) the total time of flight of electrons. (2+2+2)+(2+2)

8. (a) Arrange the four basic interactions of nature in increasing order of typical time over which they act.

(b) Write down the quark composition of baryon octet.

(c) What is color hypothesis?

(d) Find the missing particles in the following interactions :

(i) $\mu^- \rightarrow e^- + \bar{\nu}_e + ...$

(ii) $\nu_e + n \rightarrow p + ...$ 2+4+2+(1+1)