2023

PHYSICS — HONOURS

Paper : CC-12

(Statistical Physics)

Full Marks : 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.* 1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) A system with constant energy and number of constituent particles has its volume increased from V_1 to V_2. Determine the ratio of microstates Ω_1 and Ω_2.

(b) A system in thermal equilibrium at temperature T, has two allowed microstates with energies O and $K_B T$. Calculate the probability of finding the system in the ground state.

(c) In how many ways can 5 indistinguishable objects be arranged in 3 boxes when there is no restriction in the number of objects in a single box?

(d) Consider a classical particle confined inside a 2-D box of length L of each side. It has energy between ϵ to $\epsilon + \Delta\epsilon$. Calculate the number of microstates in the phase space accessible to the particle.

(e) Show that for a canonical system, mean pressure $\bar{p} = \frac{1}{\beta}\frac{\partial \ln Z}{\partial V}$.

(f) Can ${}^{7}_{3}\mathrm{Li}$ form BEC? Give reason.

(g) Show with the help of a suitable diagram that the area enclosed by the phase trajectory of a simple pendulum executing small oscillations is equal to the product of total energy E and time period T of the pendulum.

(h) Three containers, each of volume V contain N particles of a classical gas, a gas of bosons and a gas of fermions. Which amongst these will have the highest pressure and why?

2. (a) Show that the relative r.m.s. fluctuation in energy of a system in canonical ensemble

$$\frac{\sqrt{\overline{E^2} - \bar{E}^2}}{\bar{E}} \propto \frac{1}{\sqrt{N}},$$

where N is the total number of particles of the system.

Hence discuss the equivalence of microcanonical and canonical ensembles.

(b) Derive an expression for entropy in terms of the probability (P_i) of finding a member system in canonical ensemble chosen at random with energy E_i. Use this expression to establish the third law of thermodynamics. (4+2)+(3+1)

3. (a) Write the expressions for Wien's distribution law and Wien's displacement law.

(b) State Stefan's law of radiation and derive it from Wien's distribution law.

(c) Show that Rayleigh-Jeans and Wien's distribution laws appear as the two extreme cases of Planck's law. 3+(1+3)+3

4. (a) Derive BE distribution formula.

(b) Deduce Planck's radiation law from the BE distribution formula.

(c) The possible energy values for a free particle in a three-dimensional box of volume V is

$$\epsilon = \frac{\pi^2 \hbar^2}{2mV^{2/3}}\left(n_x^2 + n_y^2 + n_z^2\right),$$

where, n_x, n_y and $n_z = 1, 2, 3, \ldots$

Show that the density of states of such a system as a function of energy is given by

$$D(\epsilon) = \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \epsilon^{\frac{1}{2}}.$$

4+3+3

5. (a) Derive the expression for the mean occupation number of Fermions per energy level.

(b) Plot the nature of Fermi-Dirac distribution function at $T = 0$ and for two non-zero temperatures T_1, $T_2 (T_2 > T_1)$.

(c) What is meant by 'degenerate Fermions'?

(d) Show that the Fermi energy E_F of electrons in a metal at $T = 0$ is given by $E_F = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{\frac{3}{2}}$ with symbols having their usual meanings. 3+2+2+3

6. (a) Using MB distribution, obtain the law of equipartition of energy of an ideal gas.

(b) Enumerate the possible states of a system of two identical particles, each of which can be in three possible single particle states, and for three separate situations, where they obey the MB, BE, and FD statistics, respectively. In each case, find the ratio of probability that they are found in the same state to that of their occupying different states. Indicate how this can be interpreted as a statistical attraction or repulsion acting between identical particles. 5+(3+2)

7. (a) Determine the partition function, internal energy and entropy for a two-level system whose energy is either $-\Delta/2$ or $\Delta/2$. Discuss the low temperature and high temperature behaviour of entropy of the system.

(b) A system with just two energy levels is in thermal equilibrium with a heat reservoir at temperature 600 K. The energy gap between the levels is 0.1 eV. Find (i) the probability that the system is in the higher energy level and (ii) the temperature at which this probability will equal 0.25. (4+3)+3