2023

PHYSICS — HONOURS

(Syllabus : 2018-2019 and 2019-2020)

Paper : CC-8

(Mathematical Physics - III)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function, then show that $\vec{\nabla}_u . \vec{\nabla}_v = 0$ at every point.

(b) Find the nature of singularity at $z = 1$ for $f(z) = \dfrac{\sin^2 z}{\sqrt{z-1}}$.

(c) Two events are in space-like separation. Explain why these two events cannot be causally connected.

(d) Express the inner product between two four-vectors A^μ and B^μ using metric tensor. Hence show that $A^\mu B_\mu = A_\mu B^\mu$.

(e) Consider a decay process $A \longrightarrow B + C$, where a particle A of mass m_A is decaying into B and C with masses m_B and m_C respectively. "The decay process will only occur for $m_A \geq m_B + m_C$". — Explain.

(f) Lagrangian of a particle of mass m moving under central force is

$$L = \frac{1}{2} m \left\{ \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2 \right\} - V(r).$$

Identify the cyclic coordinate and find out corresponding conserved momentum.

Or, **(for 2018-2019 Syllabus only)**

Find out Fourier transform of the Dirac delta function in momentum space $\delta(p - p_0)$.

(g) Lagrangian of a dynamical system is time independent $\dfrac{\partial L}{\partial t} = 0$. Show that $\dfrac{dH}{dt} = 0$.

Or, **(for 2018-2019 Syllabus only)**

If $g(k)$ is Fourier transformed function of $f(x)$, then what will be the Fourier transformed function of $f(\lambda x)$?

2. (a) Let $z^5 = 1$. What will be the roots of this equation?

(b) Establish Cauchy-Riemann conditions for an analytic function in polar coordinates.

(c) Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent series for $|z-1| > 1$. Find the nature of the singularity at $z = 1$ and the residue there.

2+3+(2+1+2)

3. (a) Evaluate $\oint \frac{z^2 \exp(z)}{z^2+4} dz$ over the contours (i) $|z - i/2| = 1$ and (ii) $|z| = 3$. Mention the nature of the singular points of the integrand.

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{(5-4\cos\theta)^2}$ using Residue theorem.

(3+2)+5

4. (a) A circular ring in X-Y plane moves along X-axis with respect to an inertial frame S with speed V. Show that the shape of the ring will be ellipse relative to S frame.

(b) S' frame is moving with velocity V along X-axis with respect to S. Light is moving in X'-Y' plane keeping angle 45° with respect to X'-axis in S' frame with speed c. Using transformation of velocity components under Lorentz transformation show that speed of light with respect to S frame will also be c.

(c) A particle of rest mass m_0 and relativistic kinetic energy $3m_0c^2$ strikes and sticks to a stationary particle of rest mass $2m_0$. What is the rest mass of the composite particle?

3+4+3

5. (a) Four velocity is defined as $V^\mu = \frac{dx^\mu}{d\tau}$, where τ is proper time. Four acceleration is defined as $a^\mu = \frac{dV^\mu}{d\tau}$. $V^\mu V_\mu = c^2$ [for Minkowski metric Diag(1,–1,–1,–1)]. Show that four acceleration is orthogonal to four velocity : $V^\mu a_\mu = 0$. Hence show that four acceleration is a space-like vector.

(b) Two events in an inertial frame S are (ct_1, x_1, y_1, z_1) and (ct_2, x_2, y_2, z_2). They are in time-like separation with $t_1 > t_2$. Show that this temporal order remains invariant in any inertial frame $S' : t'_1 > t'_2$.

(c) A four vector is given by $A = (5, 0, 0, 2)$. Is it time-like? What will be its components in a frame S' which moves with velocity v along x-axis? Use metric diagonal $(1, -1, -1, -1)$.

(2+2)+3+3

6. (a) Consider two points on the X-Y plane. Express the length of a curve joining the two points and lying on the X-Y plane. Hence obtain the path of the shortest length using calculus of variation.

(b) Consider Lagrangian L of an N particle system. Show that total momentum of this system will be conserved if the Lagrangian remains invariant under translation $\vec{r}_i \longrightarrow \vec{r}_i + \vec{d}$ (Homogeneity of space), here $\vec{d}$ is an arbitrary small constant translation vector.

(c) Hamiltonian $H = H(q_i, p_i, t)$ and the canonical equations of motion :

$\dot{q}_i = \dfrac{\partial H}{\partial p_i}$ and $\dot{p}_i = -\dfrac{\partial H}{\partial q_i}$. Using Legendre transformation find out Lagrangian $L = L(q_i, \dot{q}_i, t)$.

(1+3)+3+3

Or, (for 2018-2019 Syllabus only)

(a) Let $g(k)$ is Fourier transformed function of $f(x)$. Show that $\int_{-\infty}^{\infty} |f(x)|^2\, dx = \int_{-\infty}^{\infty} |g(k)|^2 dk$.

(b) Consider a function $f(x) = 1$ in the range $-a < x < a$ and zero eleswhere. Find Fourier transformed function of $f(x)$.

(c) Find the Fourier transform of $e^{-|x|}$. 3+3+4

7. (a) Write down Lagrangian of a spherical pendulum considering point of suspension as zero potential. Find out the constants of motion.

(b) Two bobs of equal mass are connected by a horizontal spring. Two other springs are connected to the bobs and their other ends are fixed in the walls in opposite directions. It will form a coupled oscillator (disregard gravity). Write down the Lagrangian of this coupled oscillator. Determine the Hamiltonian (consider force constants of all three springs k.).

(c) A hoop is rolling without slipping along an inclined plane with its axis remaining horizontal. What are the constraints and which generalized coordinates will you use? Write down the Lagrangian in presence of gravity. (1+2)+4+(1+1+1)

Or, (for 2018-2019 Syllabus only)

(a) Poisson distribution is $f(x) = \dfrac{e^{-\lambda} \lambda^x}{x!}$. Show that mean and variance of the distribution is λ.

(b) A six sided die is thrown five times. What is the probability that a "SIX dotted face" will appear exactly three times?

(c) Show that mean value $\bar{x}$ is zero for the Gaussian distribution $\dfrac{e^{-\left(\frac{x^2}{2\sigma^2}\right)}}{\sqrt{2\pi\sigma^2}}$. 4+3+3
