2023

# PHYSICS — HONOURS

## Paper : CC-10

## (Syllabus : 2019-2020)

## [Quantum Mechanics]

## Full Marks : 50

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer ***question no.*** 1 and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Plot the normalized wave functions for the ground state $\psi_0$ and the first excited state $\psi_1$ of a quantum linear harmonic oscillator.

(b) What is the significance of zero point energy of quantum linear harmonic oscillator?

(c) Why the energy eigenfunctions of a one-dimensional harmonic oscillator with potential $V(x) \propto x^2$ are either even or odd under parity?

(d) The complete wave function of a hydrogen-like atom in a particular state is given as $\psi(r,\theta,\phi) = Nr^2 \exp\left[-\frac{zr}{3a_0}\right]\sin^2\theta e^{i2\phi}$. Determine the eigenvalue of $\hat{L}_z$, the third component of angular momentum operator.

(e) Calculate Landé g-factor for the state ${}^2P_{3/2}$ of the Hydrogen atom.

(f) Using vector atom model, find the possible values of the total angular momentum for $L = 3,\ S = \frac{3}{2}$.

(g) Find the term symbol of the ground state of the Fluorine ($Z = 9$) atom.

2. A particle in infinite square well has its initial wave function as an equal mixture of two stationary states :

$$\psi(x,0) = A[\psi_1(x) + \psi_2(x)]$$

Assume $$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \text{ and } E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

(a) Normalize $\psi(x, 0)$

(b) Find $\psi(x, t)$ and $|\psi(x, t)|^2$

(c) Compute $\langle x \rangle$. What are the angular frequency and amplitude of oscillation?

(d) Find $\langle H \rangle$. 2+3+4+1

Please Turn Over

3. (a) Consider a particle in its $n$-th excited state of the 1D quantum harmonic oscillator potential $V(x)=\frac{1}{2}m\omega x^2$ with the symbols are of usual meaning.

(i) The total energy for a harmonic oscillator is equally distributed among the kinetic and potential energy parts on an average. — From this consideration, estimate the average potential energy $\langle V(x)\rangle$ in this state. Hence find $\langle x^2\rangle$ in this state.

(ii) Using symmetry properties of the solutions, find $\langle x\rangle$ in the state.

(iii) From the expression of $\langle x\rangle$ and $\langle x^2\rangle$ obtained above, find $\Delta x$.

(iv) From the standard uncertainty relation, obtain $\Delta p$.

(b) For a quantum linear harmonic oscillator, if $\psi_0$ and $\psi_1$ are the ground state and first excited state, respectively, then show that (i) $\psi_0$ and $\psi_1$ are orthogonal and (ii) $\langle\psi_0|x|\psi_1\rangle=\frac{1}{\sqrt{2}}$.

(2+2+1+1)+(2+2)

4. (a) The ground state wave function for the Hydrogen atom is $\psi(r,\theta,\phi)=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$, where $a_0$ is the Bohr radius.

(i) Plot the radial probability density indicating the location of maximum.

(ii) Calculate the probability of finding the electron within a sphere centered at the nucleus of radius $a_0$.

(iii) Find the expectation value of $\frac{1}{r^2}$.

(b) Find $\left[\hat{L}_x,\hat{z}\right],\left[\hat{L}_x,\hat{p}_z\right]$.

(2+3+2)+3

5. (a) (i) Let $\psi_{lm}$ be the eigenstate of $\hat{L}^2$ and $\hat{L}_z$ with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$ respectively. Show that $\phi=\left[\hat{L}_x+i\hat{L}_y\right]\psi_{lm}$ is also an eigenstate of $\hat{L}^2$ and $\hat{L}_z$. Determine the eigenvalues.

(ii) Show that if $l=0$, the state $\psi_{lm}$ is also an eigenstate of $\hat{L}_x$ and $\hat{L}_y$.

(b) Mention the corrections needed in the Hamiltonian of a Hydrogen atom problem to obtain fine structures.

(4+3)+3

6. (a) Calculate the Landé g-factor for $^3S_1$ and $^3P_1$ levels. Hence, estimate the energy splitting of the two levels if a magnetic field of 1T is applied. How many spectral lines will arise from the Anomalous Zeeman splitting due to transition between these levels? Draw a neat diagram showing these transitions. [1 Bohr magneton = $9.27\times10^{-24}$ J/T]

(b) Consider a two electron system with $l_1 = 2$ and $l_2 = 1$. What are the possible angular momenta $J$ assuming $L-S$ coupling? Write the spectral term for each state. (2+2+3)+3

7. (a) What is space quantization? Explain with a suitable diagram. How can you verify this from Stern-Gerlach experiment?

(b) Show that antisymmetrized wave function for a two-particle system naturally follows Pauli's exclusion principle.

(c) State Hund's rules. (1+2+3)+2+2