

2024

MATHEMATICS — GENERAL

Paper : GE/CC-4

Full Marks : 65

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**The symbols and notations have their usual meanings.*

Group - A

1. Choose the correct alternative :

1×10

(a) The inverse element of x in a group of integers under the composition $*$ defined by $x * y = x + y + 1$ is(i) $-x$ (ii) $-x - 2$ (iii) $-x - 1$ (iv) $\frac{1}{x}$.(b) The eigenvalues of the square matrix $\begin{bmatrix} 2 & 4 & 3 \\ 4 & 5 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ are

(i) all purely imaginary or zero

(ii) all real

(iii) all purely imaginary

(iv) none of these.

(c) The set $\{(a+b\sqrt{2}) : a, b \in \mathbb{R}\}$ is a subfield of(i) $\mathbb{Q}$ (ii) $\mathbb{R}$ (iii) $\mathbb{Z}$ (iv) $\mathbb{N}$.(d) If $(2, 3) = \alpha(1, 1) + \beta(-1, 1)$ then values of α and β are

(i) 2, 3

(ii) $\frac{5}{2}, \frac{1}{2}$ (iii) $2, \frac{3}{2}$ (iv) $\frac{3}{2}, \frac{1}{2}$.(e) Hexadecimal number corresponding to the binary number $(11010101.1111)_2$ is(i) $(C5 \cdot F)_{16}$ (ii) $(D5 \cdot F)_{16}$ (iii) $(E5 \cdot F)_{16}$ (iv) $(D6 \cdot F)_{16}$.

Please Turn Over

- (f) FORTRAN expression of $\cos^{-1} x + \sqrt{x^2 - 1}$ is
- (i) $\text{COS}(1/x) + \text{SQRT}(x * x - 1)$ (ii) $\text{ACOS}(x) + \text{SQRT}(x ** 2 - 1)$
 (iii) $\text{COS}(1/x) + \text{SQRT}(x \wedge 2 - 1)$ (iv) $\text{ACOS}(x) + \text{SQRT}(x \wedge 2 - 1)$.
- (g) Which of the following is not an operating system?
- (i) DOS (ii) UNIX
 (iii) WINDOWS (iv) FORTRAN.
- (h) The mean of the binomial distribution $\left(16, \frac{1}{4}\right)$ is
- (i) 3 (ii) 7.5
 (iii) 4 (iv) 1.5.
- (i) What is the probability of getting a sum of 7, when two dice are thrown
- (i) $\frac{1}{6}$ (ii) $\frac{2}{3}$
 (iii) $\frac{1}{3}$ (iv) $\frac{7}{36}$.
- (j) For what value of ' β ' the function $f(x)$ given by
- $$f(x) = \beta e^{-2x} \text{ for } x \geq 0$$
- $$= 0, \text{ elsewhere}$$
- is a probability density function?
- (i) 2 (ii) 6
 (iii) 7 (iv) $\frac{1}{2}$.

Group - B**Unit - 1****(Algebra - II)**

2. Answer *any three* questions :

- (a) Let G be the set of real numbers except the integer 1. Let the operation '*' in G be defined by, $a * b = a + b - ab$, for all $a, b \in G$. Show that $(G, *)$ is a group. Is it abelian? 4+1
- (b) Show that the real quadratic form of $2x^2 + y^2 - 3z^2 - 8yz - 4xz + 12xy$ is indefinite. 5
- (c) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. 5

- (d) (i) Prove that in a commutative ring $(R, +, \cdot)$
 $(x + y)^2 = x^2 + 2xy + y^2$ [$x^2 = x \cdot x$, $2x = x + x$].
- (ii) Show that if $x = x^{-1}$, for all $x \in G$, where $(G, \cdot)$ is a group, is abelian. 3+2
- (e) Prove that the set $\{(x, y, z) \in \mathbb{R}^3 : x - y + z = 0\}$ is a subspace of $\mathbb{R}^3$. Obtain the basis and dimension of this space. 3+1+1

Unit - 2

(Computer Science and Programming)

3. Answer *any four* questions :

- (a) Convert $A = (345.12)_8$ and $B = (AB.CD)_{16}$ to binary form and evaluate $A - B$ using 2's complement method. 2+3
- (b) Write a flowchart for calculating area of a triangle with given three sides are A , B and C . 5
- (c) (i) Write FORTRAN expression of $\frac{a^5 + \log_e x}{\cot x + \sqrt{x}}$.
- (ii) Suppose $A = 3$, $B = 2$, evaluate $Z = 3 * A/4 * B + 2$. 3+2
- (d) Write a FORTRAN 77/90 program to test whether a year is leap year or not. 5
- (e) Write differences between RAM and ROM. What is ASCII Code? 3+2
- (f) Write an algorithm to evaluate $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$, where n is a natural number. 5
- (g) (i) Write down three differences between a compiler and an interpreter.
- (ii) Define BIT and BYTE. 3+2

Unit - 3

(Probability and Statistics)

4. Answer *any four* questions :

- (a) Prove that $\text{Var}(X) = E(X^2) - \{E(X)\}^2$.
 Find the mean of a Poisson distribution with parameter μ . 2+3
- (b) A bag contains 8 red balls and 5 white balls. Two successive draws of 3 balls are made without replacement. Find the probability that the first drawing will give 3 white balls and the second will give 3 red balls. 5
- (c) For two variables x and y the equations of two regression lines are $x + 4y + 3 = 0$ and $4x + 9y + 5 = 0$. Identify which one is of y on x . Find the means of x and y . Find the correlation coefficient between x and y . Estimate the value of x when $y = 1.5$. 5

- (d) In order to test whether a coin is perfect, the coin is tossed 5 times. The null hypothesis of perfectness is rejected if more than 4 heads are obtained. What is the probability of Type I error? Find the probability of Type II error when the corresponding probability of head is 0.2. 5
- (e) Find arithmetic mean and standard deviation of the following distribution.

x	12	18	25	42	50	57
f	8	19	40	32	24	7

5

- (f) The mean life time of a sample of 169 light bulbs manufactured by a company is found to be 1350 hours with a standard deviation of 100 hours. Establish 90% confidence limits within which the mean life time of light bulbs is expected to lie. 5
- (g) For what value of 'k', the function $f(x)$ defined by

$$f(x) = \begin{cases} kx(1-x) & , 0 < x < 1 \\ 0 & , \text{elsewhere} \end{cases}$$

is a probability density function? Obtain $P\left(X \leq \frac{1}{2}\right)$.

2+3
