

2024

MATHEMATICS — HONOURS

Paper : CC-10

(Mechanics)

Full Marks : 65

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**Symbols in the questions have their usual meanings.*

1. Answer the following multiple choice questions with only one correct option. Choose the correct option with proper justification : (1+1)×10
- (a) A force $\vec{F} = 2\hat{i} - \hat{j} - 2\hat{k}$ acts on a particle located at $P(0, 3, -1)$. Then the moment of the force $\vec{F}$ about the axis through O along $3\hat{i} - 4\hat{k}$ is
- (i) $\frac{3}{5}$ (ii) $\frac{5}{3}$
- (iii) $\frac{3}{5}$ (iv) $-\frac{5}{3}$
- (b) A uniform ladder rests in limiting equilibrium with its lower end on a rough horizontal plane and its upper end against a smooth vertical wall. If θ be the inclination of the ladder to the vertical and μ be the coefficient of friction, then
- (i) $\mu = \tan\theta$ (ii) $2\mu = \tan\theta$
- (iii) $\mu = \cot\theta$ (iv) $\mu = \frac{1}{2} \cot\theta$
- (c) A uniform cubical box of edge d is placed on the top of a fixed sphere of radius r . The equilibrium of the box will be stable if
- (i) $d > r$ (ii) $d < r$
- (iii) $d < 2r$ (iv) $d > 2r$
- (d) Suppose a given system of coplanar forces $\vec{F}_i$ ($i=1, 2, \dots, n$) acting on a rigid body has been reduced to a single force $\vec{R}$ acting at an arbitrarily chosen point O (of the body) in the plane of the forces together with a couple $\vec{G}$. Then under a change of base point
- (i) both $\vec{R}$ and $\vec{G}$ remain invariant
- (ii) $\vec{R}$ does not remain invariant but $\vec{G}$ remains invariant
- (iii) $\vec{R}$ remains invariant but $\vec{G}$ does not remain invariant
- (iv) neither $\vec{R}$ nor $\vec{G}$ remains invariant.

Please Turn Over

- (e) A particle of mass m moves with a SHM, its position of rest being at a distance a from the centre. Then the velocity at the centre is (μ is the constant of proportionality)

(i) $\frac{1}{2}m\mu a^2$

(ii) $m\mu a$

(iii) $a\sqrt{\mu}$

(iv) $-m\mu a$.

- (f) A particle moves freely in a parabolic path given by $y^2 = 4ax$, under a force always perpendicular to its axis. Then the force varies as

(i) y^3

(ii) y^2

(iii) $\frac{1}{y^3}$

(iv) $\frac{1}{y^2}$.

- (g) A particle describes the curve $p^3 = 2ar^2$ under a force F . Then the law of force is

(i) $F \propto r^{-7/3}$

(ii) $F \propto r^{2/3}$

(iii) $F \propto r^{-2/3}$

(iv) $F \propto r^{7/3}$.

- (h) A particle moves on the curve $s = c \tan \psi$ with uniform speed v . Then the normal component of acceleration is

(i) $v \cos \psi$

(ii) $v^2 \cos \psi$

(iii) $\frac{v^2}{c} \cos \psi$

(iv) $\frac{v^2}{c} \cos^2 \psi$.

- (i) For a rectilinear motion of a particle if an impulse I changes its velocity from u to v and E is the change in kinetic energy, then

(i) $E = I \left(\frac{2u+3v}{5} \right)$

(ii) $E = I \left(\frac{u+v}{2} \right)$

(iii) $E = I \left(\frac{u+2v}{3} \right)$

(iv) $E = I \left(\frac{3u+2v}{5} \right)$.

- (j) A ball of unit mass falls from a height h upon a fixed horizontal plane with coefficient of restitution e . The velocity just after its first rebound is

(i) $\sqrt{2ghe}$

(ii) $e\sqrt{2gh}$

(iii) $2ghe^2$

(iv) $\sqrt{2ghe^3}$.

(3)

B(4th Sm.)-Mathematics-H/CC-10/CBCS

Unit - 1

(Marks : 05)

2. Answer *any one* question :

- (a) A square frame ABCD of four equal jointed rods hangs from A, the shape being maintained by a string joining mid-points of AB, BC. Prove that the ratio of the tension of the string to the reaction at C is $8/\sqrt{5}$. 5
- (b) Three forces act along the straight lines $x=0, y=a; y=0, z=a; z=0, x=a$. Find the equation of Poinsot's central axis. 5

Unit - 2

(Marks : 05)

3. Answer *any one* question :

- (a) Two uniform rods AB, BC of weights W and W' respectively are smoothly joined at B and their middle points are joined across by a chord. The rods are tightly held in a vertical plane with their ends A, C resting on a smooth horizontal plane. Show by the principle of virtual work that the tension in the chord is $(W + W') \cos A \cos C / \sin B$. 5
- (b) A thin hemispherical bowl of radius b and weight W rests in equilibrium on the highest point of a fixed sphere of radius a , which is rough enough to prevent any sliding. Inside the bowl a small smooth sphere of weight w is placed. Find the condition of stable equilibrium. 5

Unit - 3

(Marks : 10)

4. Answer *any two* questions :

- (a) Deduce the tangential and normal components of velocity and acceleration of a particle moving in a plane. 5
- (b) An engine working at a constant rate H draws a load M against a resistance R . Show that the time taken to attain the speed $\frac{H}{2R}$ is $\frac{MH}{R^2} \left(\log 2 - \frac{1}{2} \right)$. 5
- (c) A particle moves in one plane under a force which is always perpendicular and towards a fixed straight line on the path, its magnitude being $\mu \div (\text{distance from the line})^2$. If initially it be at a distance $2a$ from the line and be projected with a velocity $\sqrt{\mu/a}$ parallel to the line, then prove that the path traced out is a cycloid. 5
- (d) A particle moving under a constant velocity parallel to the axis of y and a velocity proportional to y parallel to the axis of x . Show that the path is a parabola. 5

Please Turn Over

Unit - 4

(Marks : 15)

5. Answer *any one* question :

- (a) A particle moves with a central acceleration $\mu(3u^3 + a^2u^5)$, being projected from a distance a at an angle $\frac{\pi}{4}$ with a velocity equal to that in a circle at the same distance. Prove that the time to reach

the centre of force is $\frac{a^2}{\sqrt{2\mu}} \left(2 - \frac{\pi}{2} \right)$. 7

- (b) One end of an elastic string of natural length a and modulus of elasticity $2mg$ is attached to a fixed point 'O' and the other end is tied to a particle of mass m . The particle, initially held at 'O', is allowed to fall. Find the greatest extension of the string during the motion. 7

6. Answer *any one* question :

- (a) A heavy particle slides down a rough cycloid of which the coefficient of friction is μ . Its base is horizontal and vertex downwards. Show that if it starts from rest at a point, where the tangent makes an angle θ with the horizontal and comes to rest at the vertex, then $\mu e^{\mu\theta} = \sin\theta - \mu\cos\theta$. 8

- (b) If a planet was suddenly stopped in its orbit, supposed circular, then show that it would fall into the sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. 8

Unit - 5

(Marks : 10)

7. Answer *any one* question :

- (a) (i) If two smooth spheres of masses m_1 and m_2 , moving with respective velocities u_1 and u_2 in the same direction impinge obliquely, find their velocities after impact.

- (ii) A shell of mass m is projected from a gun of mass M by an explosion which generates kinetic

energy. Prove that the initial velocity of the shell is $\sqrt{\frac{2ME}{m(M+m)}}$. 5+5

- (b) (i) If the system of particles $P_1, P_2, \dots, P_N$ with masses $m_1, m_2, \dots, m_N$, has position vectors $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N$ and velocities $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_N$ respectively, then define angular momentum of the system of particle about origin. Also state the principle of conservation of angular momentum for the system.

- (ii) A particle is projected from the origin with a velocity whose components along the axes are

$\left(0, -\frac{a}{p}, b \right)$ under a force per unit mass whose components along the axes are $(a \cos pt, a \sin pt, 0)$. Find the path of the particle (a, b, p are constant). (2+2)+6