

Write the answers to each **Group**
in a separate answer-book.

2025

MATHEMATICS — MDC

Paper : CC-1

(Calculus, Geometry and Vector Analysis)

Full Marks : 75

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Symbols have their usual meanings.

Group - A

(Calculus)

(Marks : 20)

1. Answer **any four** questions :

2×4

(a) Find $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$.

(b) $y = 2 \cos x (\sin x - \cos x)$, show that $(y_{10})_0 = 2^{10}$.

(c) Evaluate $\int_0^{\pi/2} \sin^9 x \, dx$ using the reduction formula $I_n = \frac{n-1}{n} I_{n-2}$, where $I_n = \int_0^{\pi/2} \sin^n x \, dx$.

(d) Find $\frac{dy}{dx}$, if $x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$.

(e) Find the range of x in which the function $2x^3 + 3x^2 - 36x + 2026$ is monotonically increasing.

(f) Find the length of the arc of the parabola $x^2 = 4y$ from the vertex to the point where $x = 2$.

(g) Find the area bounded by curve $y = x^2$ and the line $y = 2x$.

2. Answer **any three** questions :

4×3

(a) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, $|x| < 1$, show that $(1-x^2)y_{n+2} - (2n+3)x y_{n+1} - (n+1)^2 y_n = 0$.

(b) Find a and b such that

$$\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1.$$

Please Turn Over

(5681)

(c) Show that $x > \log(1+x) > x - \frac{x^2}{2}$, for $x > 0$.

(d) If $I_n = \int_0^{\frac{\pi}{2}} \cos^{n-2} x \sin nx \, dx$, show that $2(n-1)I_n = 1 + (n-2)I_{n-1}$.

(e) Find the perimeter of $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

(f) Find the volume of the solid obtained by revolving the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 1$, $x = 4$ about x -axis.

Group - B

(Geometry)

(Marks : 35)

3. Answer **any two** questions :

2½×2

(a) Find the angle of rotation of the axes about the origin which transforms the equation $x^2 - y^2 = 4$ to $x'y' = 2$.

(b) Find the point on the conic $\frac{14}{r} = 3 - 8 \cos \theta$ whose vectorial angle is $\frac{2\pi}{3}$.

(c) Find the equation of the sphere passing through origin and the points $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$.

(d) Show that the equation $2x^2 + 3y^2 - 8x + 6y - 12z + 11 = 0$ represents an elliptic paraboloid.

4. Answer **any five** questions :

6×5

(a) If the normal to the hyperbola $xy = c^2$ at the point $\left(ct_1, \frac{c}{t_1}\right)$ meets the curve again at the point

$\left(ct_2, \frac{c}{t_2}\right)$, then show that $t_1^3 t_2 + 1 = 0$.

(b) Show that the straight line $\frac{l}{r} = A \cos \theta + B \sin \theta$ touches the conic

$\frac{l}{r} = 1 + e \cos \theta$ if $(A - e)^2 + B^2 = 1$.

- (c) If PSQ and $PS'R$ be two focal chords of an ellipse through the foci S and S' , then show that

$$\frac{SP}{SQ} + \frac{S'P}{S'Q}$$

is independent of the position of P .

- (d) Reduce the equation $7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$ to its canonical form and determine the nature of the conic.
- (e) Find the equation of the sphere which passes through the origin and touches the sphere $x^2 + y^2 + z^2 = 56$ at the point $(2, -4, 6)$.
- (f) Find the equation of the cone whose vertex is origin and which contains the curve $x^2 - y^2 + 4x = 0, x + y + z = 3$.
- (g) Find the equation of the cylinder generated by straight lines parallel to

$$\frac{x}{1} = \frac{y}{5} = \frac{z}{-2},$$

the guiding curve being the conic $x = 0, y^2 = 6z$.

- (h) Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z.$$

- (i) Find the nature of the surface represented by the equation

$$2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z - 5 = 0.$$

Group - C

(Vector Analysis)

(Marks : 20)

5. Answer **any four** questions :

2×4

- (a) For any two vectors $\vec{\alpha}$ and $\vec{\beta}$ show that

$$|\vec{\alpha} \times \vec{\beta}|^2 + |\vec{\alpha} \cdot \vec{\beta}|^2 = |\vec{\alpha}|^2 |\vec{\beta}|^2.$$

- (b) If a particle is acted on by constant forces $\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + 5\hat{k}$ and is displaced from the point $(\hat{i} + 2\hat{j} - 3\hat{k})$ to $2\hat{i} - 3\hat{j} + 7\hat{k}$, then find the total work done by the forces.

- (c) Three forces $2\hat{i} + 3\hat{j} - 5\hat{k}$, $-5\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} - 2\hat{j} + 4\hat{k}$ act on a particle, find the resultant of the forces.

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- (d) Find the vector equation of a straight line passing through the points (2, 3, 4) and (-1, 3, -2).
- (e) Find the volume of the tetrahedron whose vertices are (1, 1, 0), (1, 0, 1), (0, 1, 1) and (1, 1, 1).
- (f) If $\vec{r} = (3t, 3t^2, 2t^3)$, find $\left[\frac{d\vec{r}}{dt} \frac{d^2\vec{r}}{dt^2} \frac{d^3\vec{r}}{dt^3} \right]$.
- (g) If $\vec{F}(t) = 3t^2\hat{i} + t\hat{j} - t^3\hat{k}$, then find $\int_1^2 \vec{F}(t) \times \frac{d^2\vec{F}(t)}{dt^2} dt$.

6. Answer **any three** questions :

- (a) $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are three non-coplanar vectors, then show that $(\vec{\alpha} - \vec{\beta}) \cdot (\vec{\beta} - \vec{\gamma}) \times (\vec{\gamma} - \vec{\alpha}) = -[\vec{\alpha} \vec{\beta} \vec{\gamma}]$. 4

- (b) Find the shortest distance between the straight lines

$$\vec{r} = \vec{\alpha} + t\vec{\beta} \quad \text{and} \quad \vec{r} = \vec{\gamma} + s\vec{\delta}, \quad \text{where} \quad \vec{\alpha} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{\beta} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \quad \vec{\gamma} = \lambda\hat{i} + 3\hat{j} + 4\hat{k} \quad \text{and} \\ \vec{\delta} = 3\hat{i} + 4\hat{j} + 5\hat{k}.$$

4

- (c) If $\vec{u} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$ and $\vec{v} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$, then show that

(i) $\frac{d}{dt}(\vec{u} \cdot \vec{v}) = -6$ at $t = 1$

(ii) $\frac{d}{dt}(\vec{u} \times \vec{v}) = 7\hat{j} + 3\hat{k}$ at $t = 1$.

2+2

- (d) If $\frac{d\vec{a}}{dt} = \vec{\omega} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{\omega} \times \vec{b}$,

show that $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{\omega} \times (\vec{a} \times \vec{b})$.

4

- (e) If $\vec{F} = xy\hat{i} - 2xy^2\hat{j} + zxy^3\hat{k}$ and $\vec{G} = 2x\hat{i} + y\hat{j} - zx^2\hat{k}$,

then find $\frac{\partial^2}{\partial x \partial y}(\vec{F} \times \vec{G})$ at (1, 1, -1).

4

- (f) If $\vec{r}(t) = (t - t^2)\hat{i} + 2t^3\hat{j} - 3\hat{k}$, find $\int_1^2 \vec{r} dt$.

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