

2025

MATHEMATICS — HONOURS

Paper : CC-14

(Numerical Methods)

Full Marks : 50

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Symbols have their usual meaning.

Unit - 1

1. Each of the following questions has four possible answers of which exactly one is correct. Choose the correct alternative : 1×10

(a) If the value of $e = 2.71828$ is replaced by 2.71937, then percentage error is

- (i) 0.04% (ii) 0.05%
- (iii) 4% (iv) $\frac{1}{2}\%$.

(b) Taking step length $h = \frac{1}{2}$, what will be the value of $\Delta^6 \left[\left(1 - \frac{x}{3}\right) \left(1 - \frac{4}{5}x^2\right) \left(1 - \frac{x^2}{3}\right) \right]$?

- (i) 1 (ii) -1
 (iii) 0 (iv) 6!.

(c) Suppose in an interpolation method we use argument set $\{-2, 0, 1\}$ and get corresponding interpolating polynomial as $\frac{1}{4}x^3 + x^2 + 1$. Which method is used here?

- (i) Newton's interpolation
- (ii) Lagrange's interpolation
- (iii) Divided Difference interpolation

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(1927)

(iv) Hermite interpolation.

(d) The expression for the first derivative from Newton's forward interpolation formula at $x = x_0$ is given by

$$(i) f'(x_0) = h \left[\Delta y_0 + \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 + \frac{1}{4} \Delta^4 y_0 + \dots \right]$$

$$(ii) f'(x_0) = h \left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \dots \right]$$

$$(iii) f'(x_0) = \frac{1}{h} \left[\Delta y_0 + \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 + \frac{1}{4} \Delta^4 y_0 + \dots \right]$$

$$(iv) f'(x_0) = \frac{1}{h} \left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \dots \right]$$

(e) If $f(x) = 3x^5 + 5$, then which one gives exact result when applied to $\int_1^2 f(x) dx$?

(i) The Trapezoidal rule

(ii) The Simpson's one-third rule

(iii) The Weddle's rule

(iv) The Simpson's $\frac{3}{8}$ th rule.

(f) If a quadrature formula $\frac{3}{2}f\left(-\frac{1}{3}\right) + kf\left(\frac{1}{3}\right) + \frac{1}{2}f(1)$, that approximates $\int_{-1}^1 f(x) dx$, is found to be

exact for quadratic polynomials, then the value of k is

(i) 2

(ii) 1

(iii) 0

(iv) -2.

(g) Order of convergence of Newton-Raphson method, when we find out the value of $\sqrt[5]{3}$, correct up to four decimal places, is

(i) 1

(ii) 2

(iii) 3

(iv) 4.

(h) For which of the functions $\phi(x)$ below, is $\alpha = \sqrt{5}$ a fixed point?

$$(i) \phi(x) = \frac{x}{\sqrt{5}}$$

$$(ii) \phi(x) = \sqrt{5}x$$

(iii) $\phi(x) = x^2 - 4x$

(iv) $\phi(x) = 1 + \frac{4}{x+1}$

(i) Which of the following matrices is strictly diagonally dominant?

(i) $\begin{bmatrix} -6 & 0 & 3 \\ 7 & 8 & -2 \\ 1 & -1 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} -6 & 0 & 3 \\ 5 & 8 & -2 \\ 1 & -1 & 2 \end{bmatrix}$

(iii) $\begin{bmatrix} -6 & 0 & 3 \\ 5 & 8 & -2 \\ 1 & -1 & 3 \end{bmatrix}$

(iv) $\begin{bmatrix} -6 & 0 & 3 \\ 1 & 2 & -2 \\ 1 & -1 & 8 \end{bmatrix}$

(j) The value of $y(2.5)$ obtained by solving the initial value problem

$$\frac{dy}{dx} = \frac{y-2}{x+2}, y(2) = -1,$$

by Euler's method taking $h = 0.5$, is

(i) 0.25

(ii) -0.625

(iii) -1

(iv) -1.375.

Unit - 2Answer *any one* question.

2. (a) Prove that $\Delta \log f(x) = \log \left[1 + \frac{\Delta f(x)}{f(x)} \right]$.

(b) If $f(x) = x^3$, show that the third order divided difference $f[x_0, x_1, x_2, x_3]$ is a constant. 2+33. Derive Lagrange's interpolation formula for $(n+1)$ data (x_i, y_i) , $i = 0, 1, \dots, n$. 5**Unit - 3**Answer *any two* questions.4. Deduce Newton's forward differentiation formula from Newton's forward interpolation formula for $(n+1)$ data (x_i, y_i) , $i = 0, 1, \dots, n$ (Give at least four terms). 55. Derive Simpson's $\frac{1}{3}$ rd rule using Newton's interpolation polynomial. Define degree of precision of a quadrature formula. What is the degree of precision of Simpson's $\frac{1}{3}$ rule? 3+1+1

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6. Deduce Trapezoidal rule from Newton-Cote's formula and write its error term also. Find the largest value

of spacing h , so that error in computing $\int_{1.8}^{3.4} e^x dx$ using Trapezoidal rule is less than or equal to 0.5×10^{-5} .

3+2

7. Find the values of A, B, C such that $\int_0^h f(x) dx = h \left[Af(0) + Bf\left(\frac{h}{2}\right) + Cf(h) \right]$ is exact for polynomials $f(x)$ of degree up to 2.

5

Unit - 4

Answer *any two* questions.

8. Describe the fixed point iteration method and state a sufficient condition of convergence for this method. Write down the fixed point iteration scheme for the equation $x^3 + x - 1 = 0$ asserting the convergence of the scheme. 3+2
9. Using Newton-Raphson method obtain the numerical scheme for finding $\sqrt{N}$, where N is a positive real number. Hence apply the scheme to $N = 18$ to obtain result correct to two decimals. 3+2
10. Define the order of convergence of an iteration method for computing a simple real root of a non-linear equation $f(x) = 0$. Find the order of convergence of Newton-Raphson method for computing a simple real root of a non-linear equation $f(x) = 0$. 1+4
11. Derive the secant method for finding a real simple root of an equation $f(x) = 0$. 5

Unit - 5

Answer *any two* questions.

12. Describe the Gauss-Seidel iteration method for solving a system of linear equations. State the sufficient condition of convergence for the method. What is the advantage of this method over Gauss elimination method? 3+1+1
13. Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

using Crout's LU decomposition method.

5

14. Describe the Gauss-Jordan elimination method for numerical solution of a system of n linear equations in n unknowns $AX = b$. How does it differ from Gauss elimination method? 3+2

15. Describe the power method for finding the largest eigenvalue and the corresponding eigenvector of the square matrix A of order n . 5

Unit - 6

Answer *any one* question.

16. Explain Euler's method to solve the differential equation $\frac{dy}{dx} = f(x, y)$ with initial condition $y(x_0) = y_0$. Give its geometrical interpretation. 5

17. Obtain the Picard's iteration formula for the initial value problem $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$. Use it to the problem $\frac{dy}{dx} = y - x^2, y(0) = 1$ up to the third approximation. 2+3
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