

2026

MATHEMATICS — MINOR

Paper : MN-2

(Basic Algebra)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**Symbols have their usual notation.*

Group - A

1. Answer *any five* questions :

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- (a) Show that the product of all the values of $(1+i\sqrt{3})^{\frac{3}{4}}$ is 8.
- (b) Let α, β, γ be three roots of the equation $x^3 + 2x^2 + 3x + 1 = 0$. Find the equation whose roots are $\alpha(\beta + \gamma), \beta(\gamma + \alpha), \gamma(\alpha + \beta)$.
- (c) Using Descartes Rule of Sign, state possible number of positive, negative and imaginary roots of the equation $x^5 + 2x^4 + 7x^3 + 9x^2 - 3x - 9 = 0$.
- (d) If a, b, c, d are four positive numbers, prove that $(a + b + c + d)(a^3 + b^3 + c^3 + d^3) \geq 16abcd$.
- (e) If $a + \frac{1}{a} = 2\cos\alpha$, then prove that $a^n + \frac{1}{a^n} = 2\cos n\alpha$.
- (f) The product of two roots of the equation $x^4 + x^3 + 2x^2 + 2x + 4 = 0$ is equal to the product of the other two roots. Find the roots.
- (g) Solve the equation $x^3 - 12x + 65 = 0$ by Cardan's method.
- (h) Solve the equation by Ferrari's method $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$.
- (i) Prove that $\sin\left[i\log\frac{a-ib}{a+ib}\right] = \frac{2ab}{a^2+b^2}$, where a, b are real numbers not both zero.

Group - B

2. Answer *any five* questions :

- (a) Examine if the relation
- R
- on the set
- $\mathbb{Z}$
- is an equivalence relation;

$$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : 2a + 3b \text{ is divisible by } 5\}.$$

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- (b) Solve the following system of congruences :

$$x \equiv 2 \pmod{7}, x \equiv 5 \pmod{19}, x \equiv 4 \pmod{15}.$$

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- (c) Use mathematical induction to prove the sum of first n natural number is $\frac{n(n+1)}{2}$. 5
- (d) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two mapping. If $g \circ f: A \rightarrow C$ is bijective, prove that f is injective and g is surjective. 5
- (e) (i) Find all solution of $5x \equiv 3 \pmod{19}$.
 (ii) If n is a positive integer such that $n^3 + 1$ is a prime number, show that $n = 1$. 3+2
- (f) Let $A = \mathbb{R} \setminus \{3\}$ and $B = \mathbb{R} \setminus \{1\}$. Define $f: A \rightarrow B$ by $f(x) = \frac{x-2}{x-3}$. Show that f is one to one and onto. 5
- (g) If p and $p^2 + 8$ are both prime numbers, prove that $p = 3$. 5
- (h) A relation ρ is defined on $\mathbb{Z}$ by $x \rho y$ holds if and only if $4x + 7y$ is divisible by 11 for $x, y \in \mathbb{Z}$. Show whether ρ is an equivalence relation on $\mathbb{Z}$ or not. 5
- (i) Show that the functions $f: \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R} \setminus \{1\}$ and $g: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \setminus \{2\}$ defined by $f(x) = \frac{x}{x-2}$ and $g(x) = \frac{2x}{x-1}$ are inverses to each other. 5

Group - C

3. Answer **any five** questions :

(a) Solve the following system of linear equation :

$$x_1 - 3x_3 = 8$$

$$2x_1 + 2x_2 + 9x_3 = 7$$

$$x_2 + 5x_3 = -2$$

(b) Find the rank of the matrix by reducing it to row reduced echelon form

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

(c) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, then show that $A^2 - 4A - 5I = 0$.

(d) Find a spanning set of $\mathbb{R}^3$ containing the vectors $(1, 2, 1)$ and $(2, 1, 1)$. 5

(e) Find the dimension and basis of the solution space of the system of equations :

$$x - 2y + z = 0$$

$$x - 2y - z = 0.$$

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(f) For what values of h and k , the system of equations

$$x_1 + 2x_2 + x_3 = 8, \quad 2x_1 + x_2 + 3x_3 = 13, \quad 3x_1 + 4x_2 - hx_3 = k$$

has (i) unique solution, (ii) no solution, (iii) an infinite number of solutions.

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(g) For what real values of k , the set $S = \{(k, 1, k), (0, k, 1), (1, 1, 1)\}$ form a basis of $\mathbb{R}^3$.

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(h) (i) Find the value of λ so that the matrix $A = \begin{pmatrix} 1 & 0 \\ 1 & \lambda \end{pmatrix}$ is its own inverse.

(ii) If $\alpha = (1, 1, 2)$, $\beta = (0, 2, 1)$, $\gamma = (2, 2, 4)$, determine whether β is a linear combination of γ and α . 3+2

(i) If $P = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{pmatrix}$; then find rank of $P + Q$.

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