

2026

MATHEMATICS — MINOR

Paper : MN-3

(Ordinary Differential Equation - I and Group Theory - I)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**Symbols and Notations have their usual meanings.*

Group - A

[Ordinary Differential Equation – I]

(Marks : 45)

Answer *any nine* questions.

1. (a) Find the order and degree of the differential equation

$$\left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{\frac{3}{2}} = a \frac{d^2 y}{dx^2}.$$

- (b) Obtain the differential equation of all circles each of which touches the axis of
- x
- at the origin.

2+3

2. Solve :
- $(3x + 2y - 5)dx + (2x + 3y - 5)dy = 0$
- .

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3. Solve
- $y - x \frac{dy}{dx} = 2 \left(1 + x^2 \frac{dy}{dx} \right)$
- ; given
- $y = 1$
- when
- $x = 1$
- .

5

4. Obtain the general and singular solution of
- $yp = xp^2 - p - 2, \left(p \equiv \frac{dy}{dx} \right)$
- .

5

5. Solve :
- $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x$
- .

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6. Solve
- $\frac{d^2 y}{dx^2} + y = \tan x$
- using variation of parameters.

5

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7. Use the method of Undetermined Coefficients to find a General solution : $y''+9y=x\sin(2x)$. 5
8. Solve : $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^3+\frac{dy}{dx}$. 5
9. Solve : $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4y=e^{3x}$. 5
10. Show that $y=x^m$ is a non-trivial solution of $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0$ if $m(m-1)+Pmx+Qx^2=0$. Hence find the condition for which $y=\frac{1}{x}$ is a solution of the same ordinary differential equation. 3+2
11. Solve : $\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0$. 5
 $\frac{dy}{dt}+5x+3y=0$
12. Solve : $\sin x \frac{d^2y}{dx^2} - \cos x \frac{dy}{dx} + 2y \sin x = 0$. 5
13. Obtain the singular solution : $y = px + \frac{a}{p}$ (where $p = \frac{dy}{dx}$). 5
14. Solve : $y \left\{ x(2x+1) \frac{dy}{dx} - y \left(\frac{dy}{dx} \right)^2 \right\} = 2x^3$. 5
15. Find the integrating factor of the equation $xydx + (2x^2 + 3y^2 - 20)dy = 0$, and hence solve it. 1+4
16. Solve the equation $y(2xy+1)dx + x(1+2xy+x^2y^2)dy = 0$ by substitution $xy = v$. 5

Group - B**[Group Theory – I]****(Marks : 30)**Answer *any six* questions.

17. For any two elements a and b of a group G , show that $(ab)^2 = a^2b^2$ if and only if $ab = ba$ in G . 5
18. Prove that a group of prime order is Cyclic. Give an example of a finite group which is not Cyclic, but whose all proper subgroups are Cyclic. Justify your answer. 3+2
19. Let $(G, \circ)$ be a finite cyclic group generated by a . Then prove that $\circ(G) = n$ iff $\circ(a) = n$. 5
20. If $\alpha = (1\ 2\ 3)(4\ 5)$ and $\beta = (1\ 4)(2\ 5)$, then find $\beta^{-1}\alpha\beta$. 5
21. If in a group G , $a^5 = e$ and $aba^{-1} = b^2 \forall a, b \in G$, find the order of b . 5
22. Let $G = R - \{0\}$ and $a * b = \frac{ab}{2}$, show that $(G, *)$ is an abelian group. 5
23. Show that every finite group of order less than six must be abelian. 5
24. Let $(G, \circ)$ be a group and $a, x \in G$. Then show that $O(a)$ and $O(x^{-1} \circ a \circ x)$ are same. 5
25. Show that the set $S = \{1, i, -1, -i\}$ is a commutative group with respect to the multiplication of complex numbers. 5
26. Prove that the set of rational numbers, other than 1, forms a commutative group with respect to the composition $*$ defined by $a * b = a + b - ab$ for $a, b \in \mathbb{Q} - \{1\}$. 5
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