Radiation and its Nature

1. Old Quantum Theory

1.1. Origin

Motion of large objects involving distances larger than 10^{-6}m can be explained satisfactorily by classical physics based on following laws:

i. Newton's laws of motion $F = ma$

ii. Laws of Gravitation $F = \frac{GMm}{r^2}$

iii. Coulomb's law $\frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r^2}$

iv. Lorentz Force law $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

But certain phenomena such as energy distribution in black body radiation, photoelectric effect, Compton Effect and phenomena involving distances of order 10^{-10}m cannot be explained by classical physics.

The failure of Classical physics to explain black body radiation led Max Planck to propose "Quantum hypothesis" in 1900 which is marked as the birth of quantum theory.

1.2. Black Body Radiation

A body which can absorb the entire radiations incident on it is called a perfect black body. It's co-efficient of absorption and emission is 1. According to Kirchoff's theorem, the emissive power of a blackbody is a function of temperature only. A perfect black body is an ideal conception. Lamp black and platinum black is a nearest approach.

In laboratory, an ideal blackbody may be realized by preparing a closed hollow enclosure painted by lamp black from inside and with a small orifice in its surface. Any radiation through the hole suffers multiple internal reflections. At each reflection, more than 96% of the incident beam is absorbed by lamp black resulting into total absorption of the beam which entered through the orifice. Thus this cavity has a unit absorptive power and it behaves like a blackbody. This body, when heated, emits radiation which is called the black body radiation at that temperature.

Distribution of energy spectrum:

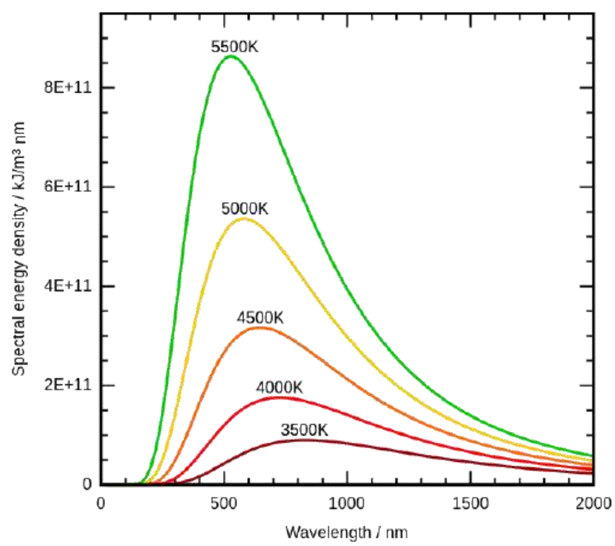

Figure: Energy distribution in Blackbody radiation. Source: internet.

The distribution of radiant energy over different wavelengths at different constant temperatures is shown in the figure. Experimental results show that-

i. E_λ for every wavelength increases with temperature.

ii. At a constant temperature, E_λ increase with λ up to a certain value of $\lambda(\lambda_m)$ and then decreases.

The shift of the peak intensity can be described by the empirical relationship known as Wien's distribution law ($\lambda_m T$=constant).

1.3. Explanation of Energy distribution in black body radiation by Classical Physics

Two well known classical laws are discussed below, none of which was fully successful to explain the nature of energy distribution in blackbody radiation.

1.3.1. Wiens Radiation Formula

Wien derived the following formula in 1896.

$$E_\lambda d\lambda = \frac{a}{\lambda^5} e^{-\frac{b}{\lambda T}} d\lambda$$

Here a and b are constants.

Limitations: For low values of λT, the formula mathches well with the experimental data. But for higher λT valus the formula gives lower E_λ values than experimental data.

1.3.2. Rayleigh Jeans Law

Rayleigh applied the principle of equipartition of energy to linear harmonic oscillator with two degrees of freedom in 1900.

The energy density or the amount of energy per unit volume inside an enclosure in the wavelength interval λ and λ+dλ is given by

$$u_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda$$

Limitations: i) This law matches the experimental data for longer wavelengths but not for shorter wavelengths.

ii) Total energy = $\int_0^\infty u_\lambda d\lambda = \infty$ which is not possible.

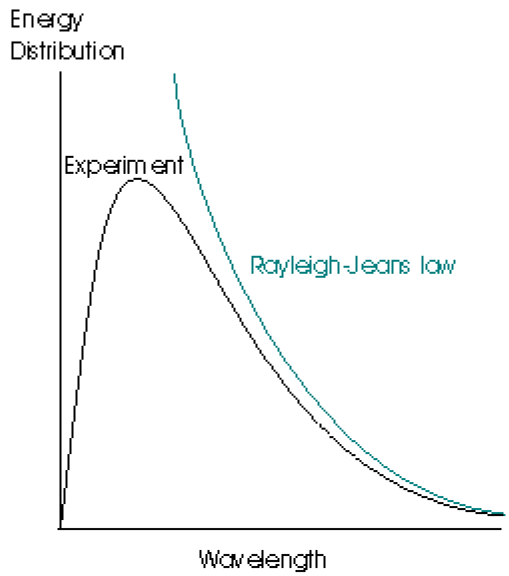

Figure: Mismatch of R-J law with experiment. Source: internet

1.4. Planck's Quantum Theory

Failure of Rayleigh Jean's law or Wien's displacement law led to a new hypothesis to explain black body radiation. Max Planck put forward a new postulate. The basic two assumptions of this theory are-

i) A simple harmonic oscillator can not have any arbitrary values of energy. They can only have finite values of total energy E that are given by

$$E = nh\nu$$

Where n=0,1,2,3,…. , ν=frequency, h=Planck Constant

This is energy quantization. The basic unit of energy is $h\nu$ and it is called a quantum of energy. It shows that energy is quantized. n is the quantum number.

ii) The emission or absorption of energy occurs only when the oscillator jumps from one energy state to another. If the oscillator jumps from a higher energy state of q.no. n_2 to a lower energy state of q.no. n_2, the energy emitted is given by

$$E_2 - E_1 = (n_2 - n_1)h\nu$$

- Einstein extended Planck's quantum theory by assuming that a monochromatic radiation of frequency ν consists of a stream of photons each of energy hν and the photons travels with the speed of light.

Planck's Radiation Law:

On the basis of quantum statistics, Planck obtained the formula for an average energy of an oscillator

$$E = \frac{h\nu}{e^{\frac{h\nu}{KT}} - 1}$$

The number of oscillations or degrees of freedom per unit volume in frequency range ν and ν+dν is given by

$$N(\nu)d\nu = \frac{8\pi\nu^2}{c^3}d\nu$$

Thus the average value of energy of various modes of oscillations in black body radiation is given by

$$U_\nu d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{KT}} - 1}d\nu$$

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda KT}} - 1}d\lambda$$

This formula agrees well with experimental results both for long and short wavelengths.

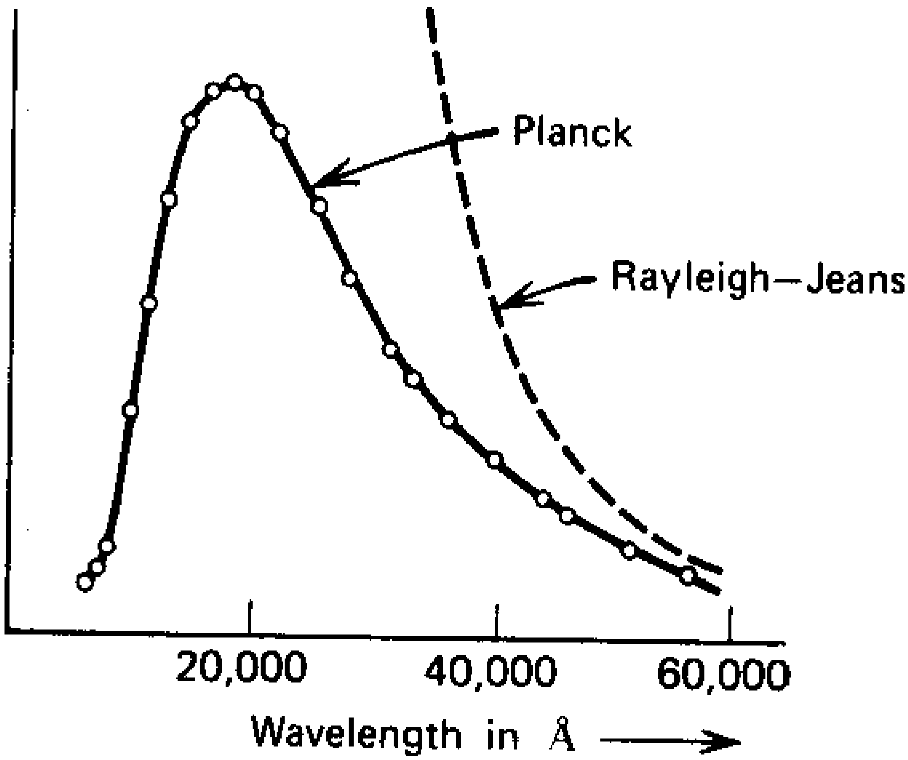

Figure: Points-Experimental data. Solid line-Planck formula.Dotted line-RJ law. Source: internet