Mass Defect & Binding Energy

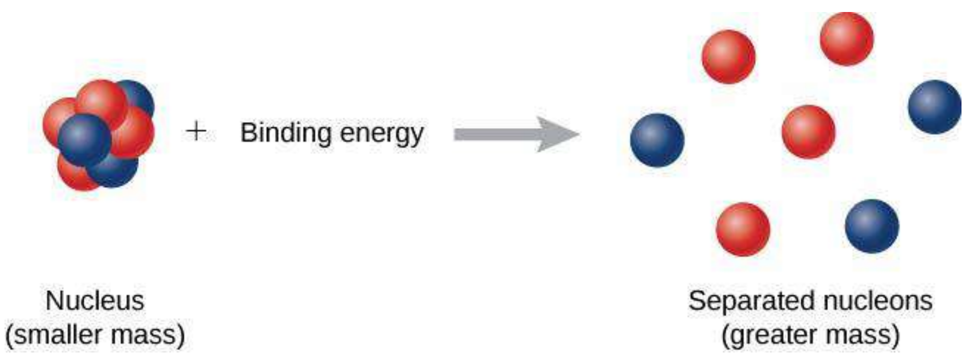
According to nuclear particle experiments, the total mass of a nucleus (m_{nuc}) is *less* than the sum of the masses of its constituent nucleons (protons and neutrons). For a nucleus with Z protons and mass number A, the mass difference or **mass defect** is given by

$$\Delta m = Zm_p + (A-Z)m_n - m_{nuc}$$

where Zm_p is the total mass of the protons, $(A-Z)m_n$ is the total mass of the neutrons, and m_{nuc} is the mass of the nucleus. According to Einstein's special theory of relativity, mass is a measure of the total energy of a system ($E=mc^2$). The energy equivalent to mass defect is alled the binding energy B of the nucleus. Thus

$$B = [Zm_p + (A-Z)m_n - m_{nuc}]\, c^2$$

The binding energy is equal to the amount of energy released in forming the nucleus.



Example 1. Calculate the mass defect and the binding energy of the deuteron. The mass of the deuteron is $m_D=3.34359\times10^{-27}$kg or 2.014102u.

Solution

For the deuteron Z=1 and A=2. The mass defect for the deuteron is

$\Delta m=m_p+m_n-m_D=1.008665$ u+ 1.007825 u- 2.014102u = 0.002388u

The binding energy of the deuteron is then

$B= \Delta m\, c^2= \Delta m \times931.5$ MeV/u=2.224MeV.

Over two million electron volts are needed to break apart a deuteron into a proton and a neutron. This very large value indicates the great strength of the nuclear force. By comparison, the greatest amount of energy required to liberate an electron bound to a hydrogen atom by an attractive Coulomb force (an electromagnetic force) is about 10 eV.

Graph of Binding Energy per Nucleon vs mass number:

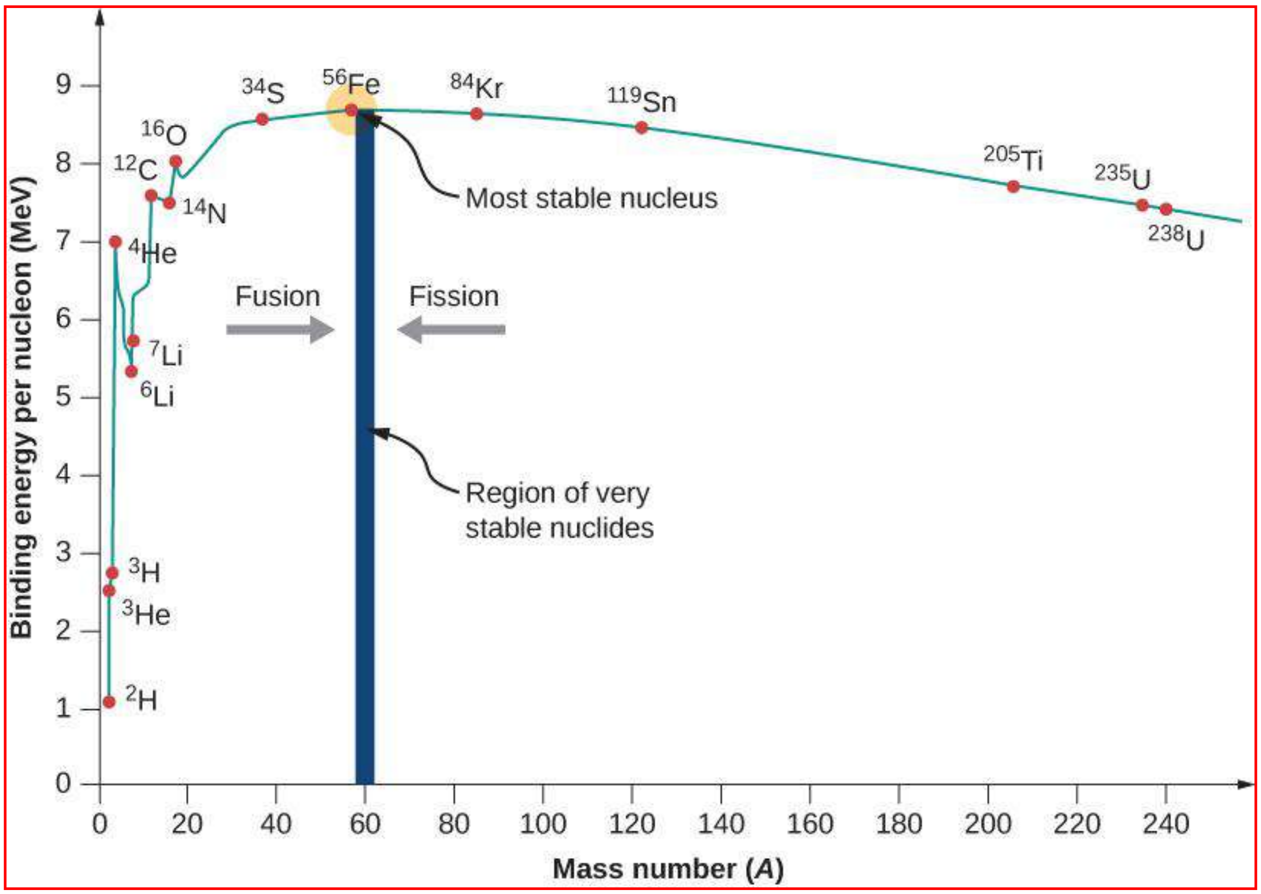
In nuclear physics, one of the most important experimental quantities is the *binding energy per nucleon* which is defined as the binding energy of a nucleus divided by the number of nucleons it contains. Figure shows the variation of average binding energy per nucleon as a function of mass number A. Some of the features are-

i) For low atomic number BE per nucleon is very low and rises as A increases. The peak at *A*=4 corresponds to the exceptionally stable $^4He_2$ nucleus, which is the alpha particle.

ii) Typical values range from 6–10 MeV, with an average value of about 8 MeV.

iii) There is maximum of about 8.8 MeV per nucleon for nuclei of mass number *A*=56 (Fe Nucleus). The peak value suggests that the iron nucleus is the most stable nucleus in nature.

iv) Beyond A=60, the curve decreases slowly with increasing A and reaches 7.6 MeV per nucleon at A=238.



Plot of Experimental data points of BE per Nucleon vs Mass number. Source: Internet

Solve the following Problems:

1. Calculate the binding energy and average binding energy per nucleon of ${}^{12}_{6}C$. Mass of ${}^{12}_{6}C$ is 12.0u. Mass of proton 1.007825 u and mass of neutron 1.008665u.
2. Compute the binding energy of last proton in a ${}^{12}_{6}C$ nucleus if the mass of this nucleus is 12u and mass of ${}^{11}_{5}B$ nucleus is 11u. Mass of proton is 1.007825 u.
3. How much energy is required to remove i) a proton and ii) a neutron from ${}^{16}_{8}O$? Mass of proton 1.007825 u and mass of neutron 1.008665u.
4. The binding energy of ${}^{35}_{17}Cl$ is 298 MeV. Find its atomic mass. Mass of hydrogen atom=1.007825u, mass of neutron 1.008665u. 1u=931.5MeV.