Radiation and its Nature

Chapter-1

Old Quantum Theory

1.1. Origin

Motion of large objects involving distances larger than 10^{-6}m can be explained satisfactorily by classical physics based on following laws:

i. Newton's laws of motion $F = ma$

ii. Laws of Gravitation $F = \frac{GMm}{r^2}$

iii. Coulomb's law $\frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r^2}$

iv. Lorentz Force law $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

But certain phenomena such as energy distribution in black body radiation, photoelectric effect, Compton Effect and phenomena involving distances of order 10^{-10}m cannot be explained by classical physics.

The failure of Classical physics to explain black body radiation led Max Planck to propose "Quantum hypothesis" in 1900 which is marked as the birth of quantum theory.

1.2. Black Body Radiation

A body which can absorb the entire radiations incident on it is called a perfect black body. It's co-efficient of absorption and emission is 1. According to Kirchoff's theorem, the emissive power of a blackbody is a function of temperature only. A perfect black body is an ideal conception. Lamp black and platinum black is a nearest approach.

In laboratory, an ideal blackbody may be realized by preparing a closed hollow enclosure painted by lamp black from inside and with a small orifice in its surface. Any radiation through the hole suffers multiple internal reflections. At each reflection, more than 96% of the incident beam is absorbed by lamp black resulting into total absorption of the beam which entered through the orifice. Thus this cavity has a unit absorptive power and it behaves like a blackbody. This body, when heated, emits radiation which is called the black body radiation at that temperature.

Distribution of energy spectrum:

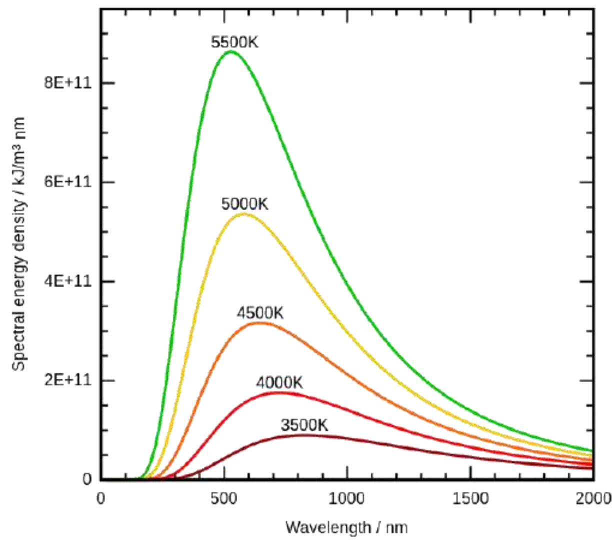


Figure: Energy distribution in Blackbody radiation. Source: internet.

The distribution of radiant energy over different wavelengths at different constant temperatures is shown in the figure. Experimental results show that-

i. E_λ for every wavelength increases with temperature.

ii. At a constant temperature, E_λ increase with λ up to a certain value of $\lambda(\lambda_m)$ and then decreases.

The shift of the peak intensity can be described by the empirical relationship known as Wien's distribution law ($\lambda_m T$=constant).

1.3. Explanation of Energy distribution in black body radiation by Classical Physics

Two well known classical laws are discussed below, none of which was fully successful to explain the nature of energy distribution in blackbody radiation.

1.3.1. Wiens Radiation Formula

Wien derived the following formula in 1896.

$$E_\lambda d\lambda = \frac{a}{\lambda^5} e^{-\frac{b}{\lambda T}} d\lambda$$

Here a and b are constants.

Limitations: For low values of λT, the formula mathches well with the experimental data. But for higher λT valus the formula gives lower E_λ values than experimental data.

1.3.2. Rayleigh Jeans Law

Rayleigh applied the principle of equipartition of energy to linear harmonic oscillator with two degrees of freedom in 1900.

The energy density or the amount of energy per unit volume inside an enclosure in the wavelength interval λ and $\lambda+d\lambda$ is given by

$$u_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda$$

Limitations: i) This law matches the experimental data for longer wavelengths but not for shorter wavelengths.

ii) Total energy = $\int_0^\infty u_\lambda d\lambda = \infty$ which is not possible.

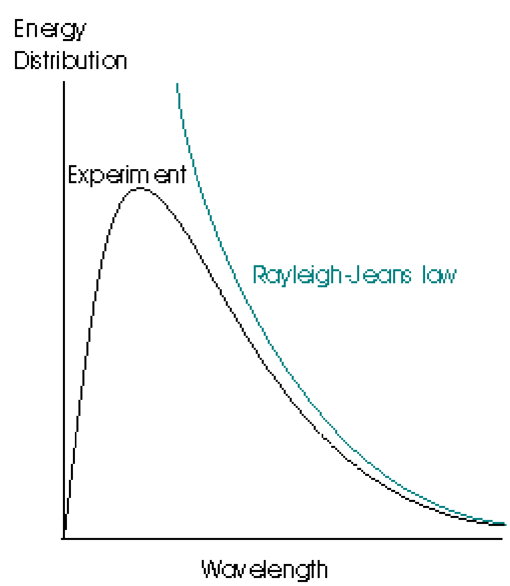


Figure: Mismatch of R-J law with experiment. Source: internet

1.4. Planck's Quantum Theory

Failure of Rayleigh Jean's law or Wien's displacement law led to a new hypothesis to explain black body radiation. Max Planck put forward a new postulate. The basic two assumptions of this theory are-

i) A simple harmonic oscillator can not have any arbitrary values of energy. They can only have finite values of total energy E that are given by

$$E = nh\nu$$

Where n=0,1,2,3,…. , ν=frequency, h=Planck Constant

This is energy quantization. The basic unit of energy is $h\nu$ and it is called a quantum of energy. It shows that energy is quantized. n is the quantum number.

ii) The emission or absorption of energy occurs only when the oscillator jumps from one energy state to another. If the oscillator jumps from a higher energy state of q.no. n_2 to a lower energy state of q.no. n_2, the energy emitted is given by

$$E_2 - E_1 = (n_2 - n_1)h\nu$$

- Einstein extended Planck's quantum theory by assuming that a monochromatic radiation of frequency ν consists of a stream of photons each of energy hν and the photons travels with the speed of light.

Planck's Radiation Law:

On the basis of quantum statistics, Planck obtained the formula for an average energy of an oscillator

$$E = \frac{h\nu}{e^{\frac{h\nu}{KT}} - 1}$$

The number of oscillations or degrees of freedom per unit volume in frequency range ν and ν+dν is given by

$$N(\nu)d\nu = \frac{8\pi\nu^2}{c^3}d\nu$$

Thus the average value of energy of various modes of oscillations in black body radiation is given by

$$U_\nu d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{KT}}-1}d\nu$$

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda KT}}-1}d\lambda$$

This formula agrees well with experimental results both for long and short wavelengths.

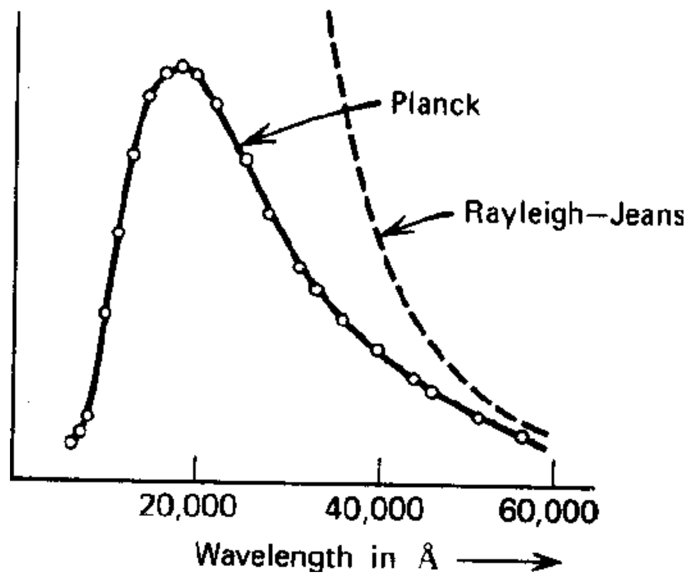
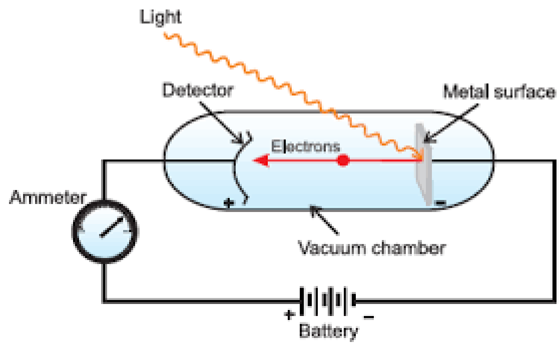


Figure: Points-Experimental data. Solid line-Planck formula.Dotted line-RJ law. Source: internet

1.5. Photoelectric Effect

When an electromagnetic radiation of sufficiently high frequency is incident on a clean metal surface, electrons are emitted from it. This phenomenon is known as photoelectric effect. The emitted electrons are known as photoelectrons.



1.5.1. Experimental Study

The photoelectric current depends on the following factors-

- **i)** Frequency of Incident Radiation
- **ii)** Intensity of Incident Radiation
- **iii)** Potential difference between electrodes
- **iv)** Nature of emitting surface

i) Frequency: The negative values of stopping potential increases with increase in frequency

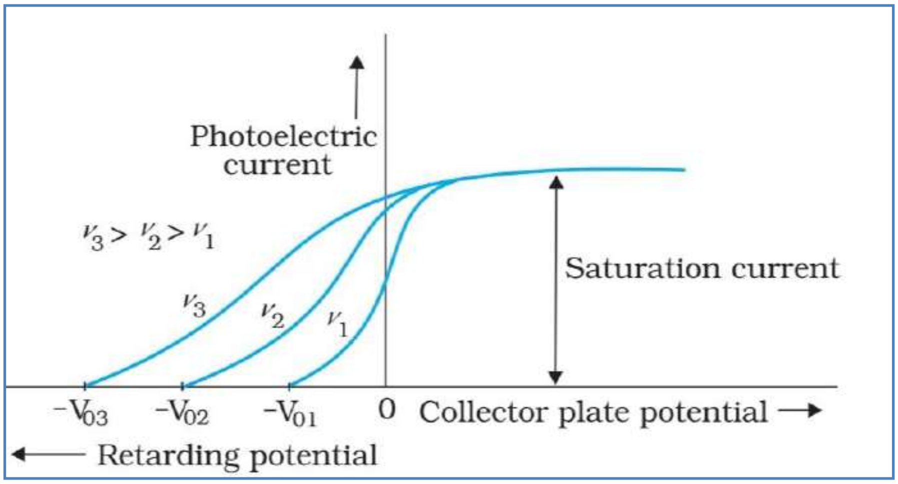


Figure: Frequency dependence of photoelectric current

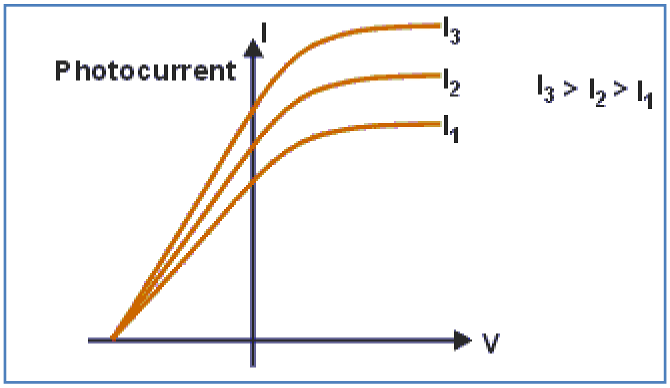


Figure: Intensity dependence of photoelectric current

ii) **Intensity:** Finite current is recorded even when the potential is zero. The negative potential at which photocurrent ultimately becomes zero is called the stopping potential. It is independent of intensity. It depends on the frequency and nature of material. The stopping potential just stops the photoelectrons having maximum kinetic energy from reaching the collecting electrode.

$$\frac{1}{2}mv_{max}^2 = eV_0$$

$$m = mass\ of\ electron, v_{max} = maximum\ velocity, e = electronic\ charge, V_0 stopping\ potential$$

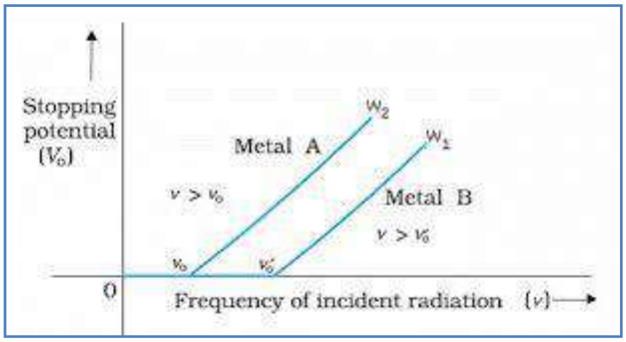


Figure: Frequency dependence of stopping potential

- ## Important Conclusions:

a) The photoelectric current depends upon the intensity of light but independent of frequency or wavelength of light.

b) Photoelectrons are emitted with all possible velocities from 0 to v_{max}. This velocity is independent of the intensity of light but depends on frequency. The maximum KE depends linearly with frequency.

c) Photoelectron emission is an instantaneous effect. There is no time gap between incidence of light and emission of photoelectrons.

1.5.2. Failure of Classical Electromagnetic theory to explain photoelectric effect

i) The existence of threshold frequency
ii) Non-dependence of maximum KE on intensity
iii) Absence of time lag

1.5.3. Einstein's explanation of photoelectric effect

In 1905 Einstein applied Planck's quantum theory and made following two assumptions:

i) A radiation of frequency ν consists of a bunch of discrete quanta each of energy hν. These are called photons. The photons move with the speed of light.

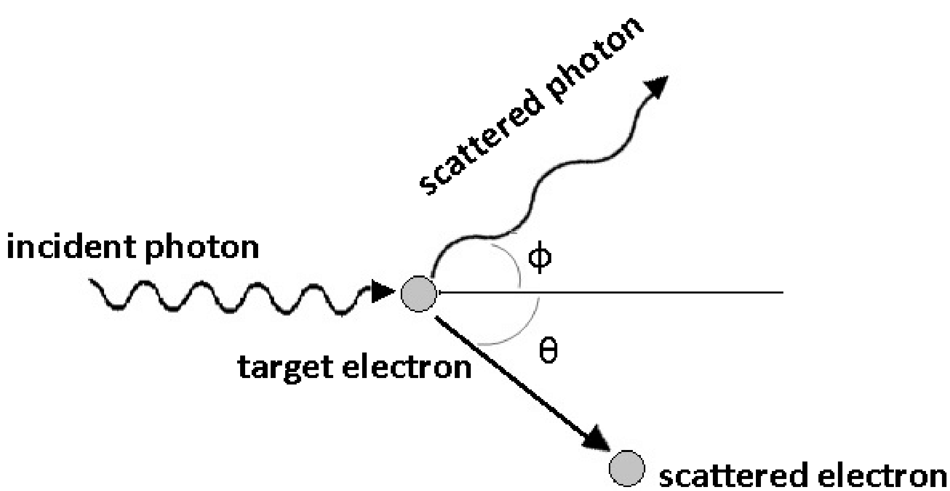
ii) When a photon of energy hν is incident on a metal surface, the entire energy is absorbed by a single bound electron without any time lag provided that hν is greater than the binding energy of the electron.

This binding energy is called the work function of the metal and this energy must be used up by an electron to leave the surface. Thus,

$$\frac{1}{2}mv_{max}^2 = h\nu - W_0 = h(\nu - \nu_0) = eV_0$$

These set of equations is known as Einstein's Photoelectric Equation.

1.6. Compton Effect



Compton in 1923 observed that when a beam of monochromatic X rays of short wavelength is incident on a lighter element such as C, B etc, then in addition to unmodified incident radiation of wavelength λ_0, the scattered beam also contains a modified radiation of slightly longer wavelength λ or lower frequency ν. This incoherent scattering of em radiation by free electron is called Compton scattering or Compton Effect.

1.6.1. Failure of classical theory:

In view of classical wave theory, X rays are electromagnetic waves which cause electrons in a target material to oscillate with the same frequency. The oscillating electrons then reradiate X rays of the same frequency. Hence the scattered radiation should consist of single frequency. Again the electrons should radiate X-rays uniformly in all possible directions and hence the wavelength of the scattered radiation should not vary with the scattering angle.

1.6.2. Explanation:

Compton applied Einstein's quantum theory of light with the assumption that incident photons possess momentum in addition to energy. Let us derive an expression for Compton shift.

Primary radiation of wavelength λ_0 or frequency ν_0 contains photons of energy $E_0=h\nu_0$ and momentum $h\nu_0/c$. Let the rest mass of electron is m_0.

Conservation of energy:

$$h\nu_0 = h\nu + (m - m_0)c^2 \dots\dots\dots\dots\dots\dots (i)$$

Here $T = (m - m_0)c^2$ = relativistic KE of recoiled electron, $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ =dynamical mass of electron.

Conservation of linear momentum:

Along X: $\frac{h\nu_0}{c} = \frac{h\nu}{c}\cos\varphi + mv\cos\theta$, or, $\frac{h}{\lambda_0} - \frac{h}{\lambda}\cos\varphi = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\cos\theta$

Along Y: $0 = \frac{h\nu}{c}\sin\varphi + mv\sin\theta$, or, $\frac{h}{\lambda}\sin\varphi = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\sin\theta$

Squaring and adding,

$$\left(\frac{h}{\lambda_0} - \frac{h}{\lambda}\cos\varphi\right)^2 + \left(\frac{h}{\lambda}\sin\varphi\right)^2 = \frac{(m_0 v)^2}{1-\frac{v^2}{c^2}}$$

$$or, \frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2}\cos^2\varphi - \frac{2h^2}{\lambda\lambda_0}\cos\varphi + \frac{h^2}{\lambda^2}\sin^2\varphi = \frac{(m_0v)^2}{1-\frac{v^2}{c^2}}$$

$$or, \frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda_0}\cos\varphi = \frac{(m_0v)^2}{1-\frac{v^2}{c^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

From eqn (i), substituting ν=c/λ, we get

$$\frac{h}{\lambda_0} - \frac{h}{\lambda} + m_0c = mc$$

$$\frac{h}{\lambda_0} - \frac{h}{\lambda} + m_0c = \frac{m_0c}{\sqrt{1-\frac{v^2}{c^2}}}$$

Squaring both sides, $\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda_0} + m_0^2c^2 + \frac{2m_0ch}{\lambda_0} - \frac{2m_0ch}{\lambda} = \frac{m_0^2c^2}{1-\frac{v^2}{c^2}} \ldots\ldots\ldots(iii)$

Substituting (iii) from (ii),

$$\frac{2h^2}{\lambda\lambda_0}(1-\cos\varphi) - 2m_0ch\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) = \frac{m_0^2(v^2-c^2)}{1-\frac{v^2}{c^2}} + m_0^2c^2$$

$$\frac{1}{\lambda_0} - \frac{1}{\lambda} = \frac{h}{\lambda\lambda_0 m_0 c}(1-\cos\varphi)$$

$$\therefore \Delta\lambda = \lambda - \lambda_0 = \frac{h}{m_0c}(1-\cos\varphi)$$

$\Delta\lambda$ is called the Compton shift and it is independent of wavelength of the incident beam and the nature of the target but depends on the scattering angle φ.

$$\varphi = 0^o, \Delta\lambda = 0$$

$$\varphi = 90^o, \Delta\lambda = \frac{h}{m_0c}$$

$$\varphi = 180^o, \Delta\lambda = \frac{2h}{m_0c}$$

$$\frac{h}{m_0c} = constant = \boldsymbol{Compton\,Wavelength}$$

Compton wavelength is the wavelength of photon having energy=rest mass energy of electron.

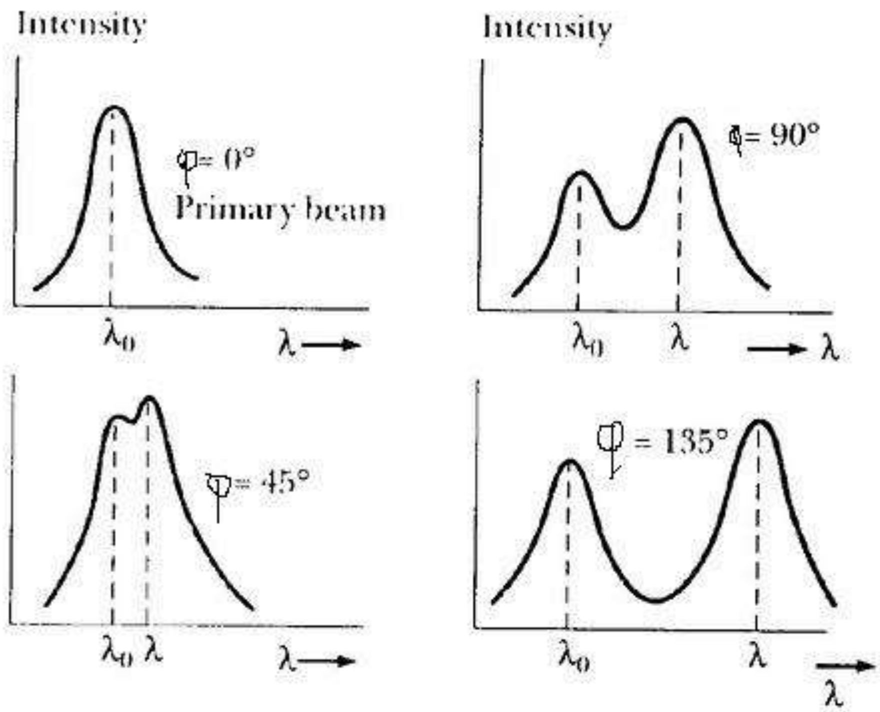


Figure: Result of the Compton scattering experiment

1.7. Stability of an atom

1.7.1. Rutherford Model

An atom consists of very small sphere of radius 10^{-14}m in which positive charge and all its mass is concentrated in the centre called nucleus. Electrons revolve round this nucleus in circular orbits of radius 10^{-10}m. The positive charge of nucleus is equal to the negative charge of electrons. The centripetal force for circular motion of electrons is provided by the electrostatic force of attraction between nucleus and electrons.

1.7.2. Failure of this model

According to the classical electromagnetic theory an accelerated charge particle radiates energy in the form of EM radiation. Due to this loss of energy electrons would move in a spiral path with decreasing radius and ultimately it would fall on nucleus. In this process of radiating continuous energy this should give rise to continuous spectrum and electrons would collapse on nucleus in 10^{-8}s.

- But atoms are stable.
- Atoms emit radiation of discrete wavelength giving rise to line spectrum.

Thus Rutherford model can't explain the stability of an atom.

1.8. Bohr's quantization of energy levels

Only those orbits are stable for which the magnitude L of the electron's orbital angular momentum is quantized: being equal to an integral multiple of $\frac{h}{2\pi}$.

$$L = mvr = n\frac{h}{2\pi} = n\hbar \; (n = 1,2,3, \dots)$$

Bohr's theory of Hydrogen atom:

1. The electron in the Hydrogen atom revolves in a stable circular orbit round the nucleus under the action of Coulomb's force of attraction between positively charged nucleus and negatively charged electron.
$$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$
2. In an allowed stable orbit the magnitude L of the orbital angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$.
$$L = mvr = n\frac{h}{2\pi} = n\hbar \; (n = 1,2,3, \dots)$$
3. Transition of electrons can only occur from one stationary orbit to another. The energy emitted is the difference of energies between the stationary states.
$$h\nu = E_{n_2} - E_{n_1}$$

1.9. Limitations of Old quantum Theory:

The quantum theory based on Bohr's quantum condition and Wilson-Sommerfield quantum rule is known as old quantum theory. It is unable to explain some problems like energy states of hydrogen atom, harmonic oscillator problem etc. The limitations of the old quantum theory are as follows:

i) It is not applicable to non-periodic systems.
ii) It cannot explain the relative intensities of the spectral lines.
iii) The spectral lines of Hydrogen molecule and Helium atom cannot be explained.
iv) The theory does not account the spin motion of electron and Pauli Exclusion Principle.