Particle Accelerators

Particle accelerators are used to accelerate, focus and project subatomic particles using electromagnetic fields.

***Example**:* Large Hadron Collider in Geneva [Largest], SuperKEKB in Japan, RHIC in New York, Tevatron in FermiLab

There are two types of accelerators:

A. Electrostatic Accelerators

It uses electrostatic fields to accelerate particles. Ex. Cockcroft Walton Generator, Van de Graff Generator.

B. Electrodynamic or Electromagnetic accelerators

It uses changing electromagnetic fields to accelerate particles. Ex. Cyclotron, Betatron.

Need of an accelerator:

Accelerators are needed in nuclear physics experiments for-

i) Nuclear Reaction
ii) To produce isobars or isotopes
iii) To get information about nucleus and its excited states

Size of a nucleus~ 10^{-14} m. To make a nuclear reaction or to pass to the interior of the nucleus, the particle de Broglie wavelength should be less or nearly equal to nuclear radius. So the momentum of the accelerating particle must be greater than h/R.

$$p > \frac{h}{\lambda} \; or \; \frac{h}{R}$$

$$= \frac{6.67 \times 10^{-34}}{10^{-14}}$$

$$\cong 10^{-20} kg.m/s \; (high)$$

Thus the Kinetic Energy of the projectile (such as α) $T > \frac{p^2}{2m} \sim 20 \; MeV$

Particles with kinetic energy T are produced in an accelerator. Now we discuss some typical accelerators that are used in particle physics experiments.

1. Linear Accelerator

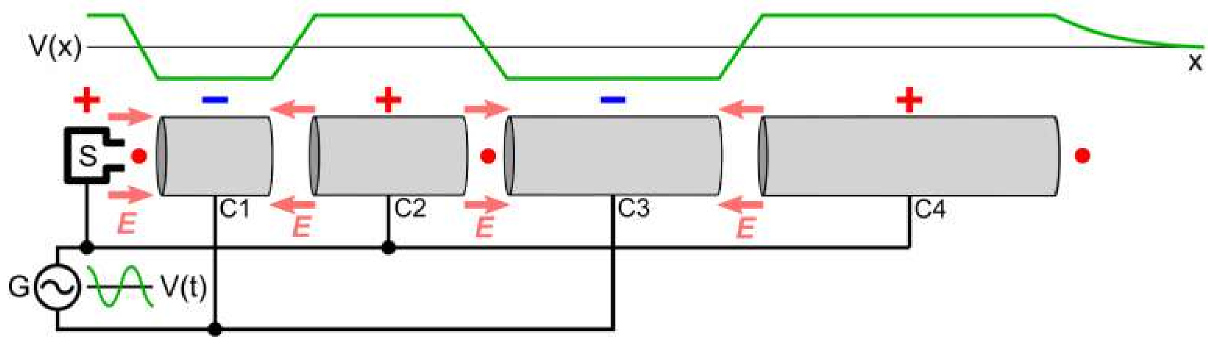
Gustav Ising (1924), Rolf Wideroe (1928)

A linear particle accelerator (linac) accelerates in a straight line with the target fixed at one end.

1.1. Basic requirements:

i) Charged Particle ii) E field

KE~ 5-10MeV, Freq~ 10^{4-5}Hz, V~2-5KV



Graphical image of a linear accelerator; Source: Wikipedia

1.2. Construction:

- A straight hollow pipe with high vaccum so that accelerated particles do not collide with air molecules
- S: Particle source
- C1,C2,C3,C4: Open ended cylindrical electrodes
- G: Oscillator that generates RF AC voltage

1.3. Uses:

- Radiation Therapy
- Medical uses
- Medical isotope development

1.4. Advantages:

- Higher particle energies than electrostatic accelerators

1.5. Disadvantages:

- Device length is limited
- Great number of associated devices
- Cavities get heated

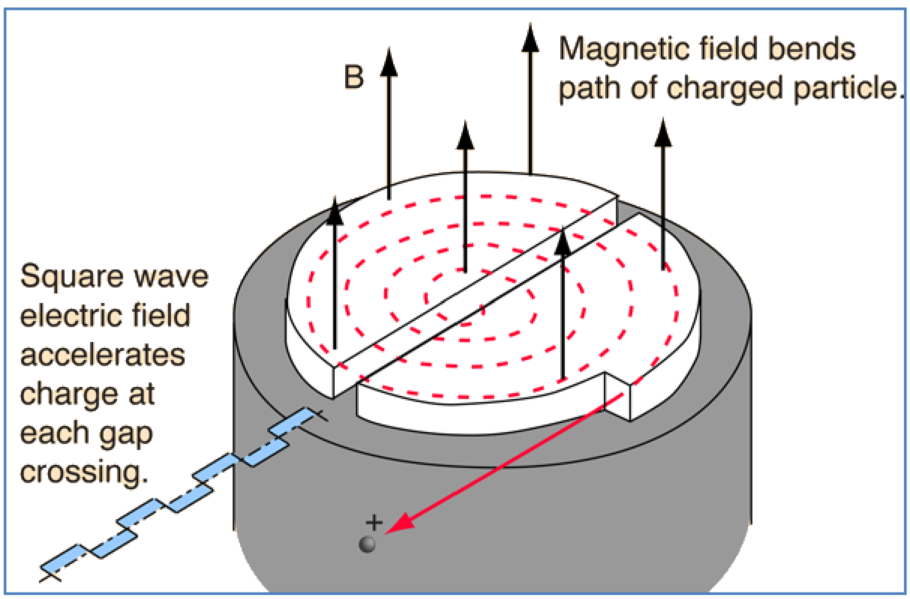
2. Cyclotron

Lawrence and Livingstone, 1931

A cyclotron is a circular accelerator in which various kinds of charged particles can be accelerated. It uses a magnetic field to maintain the circular path of the charged particles.

2.1. Construction:

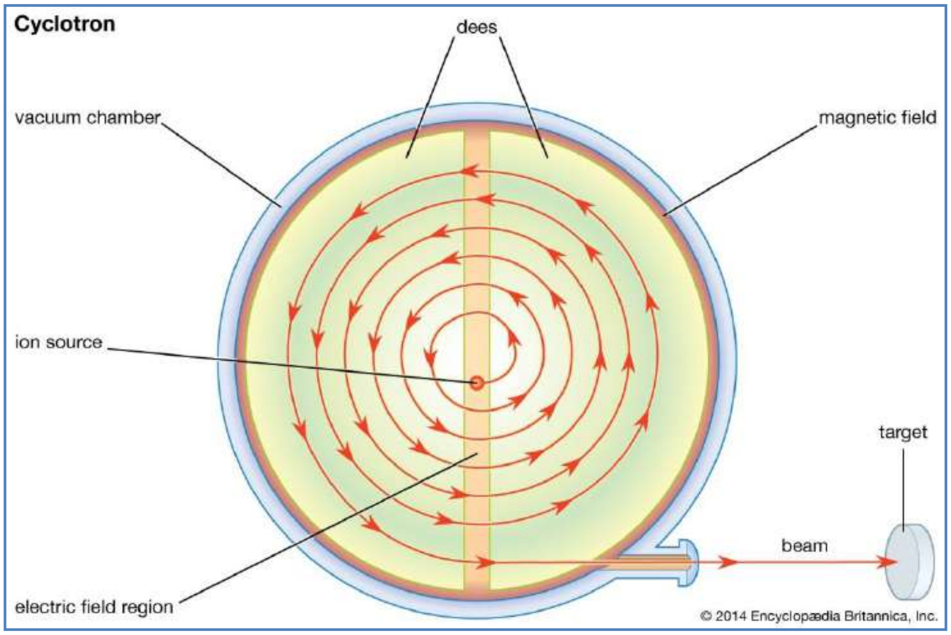
- It consists of a large evacuated cylindrical box between the pole pieces of an electromagnet.
- Inside the box two hollow D shaped metal cavities called the 'dees' are kept.
- Open side of dees is slightly displaced from each other.
- The dees are connected to a high frequency ac supply.
- Electric field inside the dees is zero due to shielding effect.
- The source is kept in between the dees.
- Magnetic field is perpendicular to the dees.



2.2. Working Principle:

- Due to ac voltage, when one dee is positive, the other is negative.
- A positive charged ion is emitted from the source and let D_1 is positive, D_2 is negative.
- The ion travels to D_2, accelerated by electric field and enters electric field free region inside D_2.
- Because of the magnetic field, ion moves in a circular path.
- AC frequency is so adjusted that when ion coming out of D_2, D_1 becomes negative.

- The ion is again accelerated and moves in a circular path inside D_1.
- Thus, at each instant, the ion accelerates and gains energy from electric field.
- As the velocity of ion increases, radius of the path also increases.
- At the end, the ion beam reaches the outer rim and deflected towards the target.



2.3. Calculation of frequency of alternating field and maximum energy of ion:

Let the charge of ion= +q, mass of ion= m, magnetic field=B, speed of ion=v, radius of circular path=r.

Now, Magnetic force provides the centripetal force.

$$qvB = \frac{mv^2}{r}$$

$$v = \frac{qBr}{m}$$

This equation is known as resonance condition. Time period of oscillation

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

The frequency of revolution is given by

$$f = \frac{qB}{2\pi m}$$

This is known as cyclotron frequency. So the frequency of the oscillating voltage needed depends on-

i) q/m ratio of the ion
ii) applied magnetic field

The maximum kinetic energy of the ion

$$KE_{max} = \frac{1}{2}mv_{max}^2 = \frac{q^2B^2R^2}{2m}$$

R is the maximum radius. In terms of the frequency of the oscillating field,

$$KE_{max} = 2\pi^2 f^2 R^2 m = qV$$

Where V is the equivalent potential difference.

Problem: A proton is rotating in a cyclotron of radius 0.8m under a magnetic field 0.6 Tesla. Find the frequency needed for resonance and maximum kinetic energy of the proton.

Answer:

$$f = \frac{0.6 \times 1.6 \times 10^{-19}}{2\pi \times 1.672 \times 10^{-27}} \cong 10^7 Hz$$

$$T_{max} = 2\pi^2 \times 1.6 \times 10^{-27} \times 10^{14} \times 0.8^2 J = 11 MeV$$

2.4. Drawbacks:

- The Cyclotron machine operates successfully for heavy ions (α, p, Li^{++}) but not suitable to accelerate lighter charged particles like electron. The maximum energy obtained in a cyclotron is approximately 20MeV for protons, 40MeV for Deuterons, 80MeV for α particle but <<0.01MeV for electrons. So electrons are not suitable.
- $\omega = \frac{Bq}{m}$, a constant for a no-relativistic particle. But for a relativistic particle, its dynamical

mass $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, $\omega = \frac{Bq}{m_0}\sqrt{1-\frac{v^2}{c^2}}$

So ω decreases with the speed of the particle and the period of ion revolution increases with velocity or radius. The ion at high velocity takes longer time to make half revolution than the oscillating field. Ultimately particle lags in phase of oscillating voltage, and finally it is no longer accelerated.

For electron, rest mass is only 0.51MeV. It reaches approx the speed of light (~0.98c) at energy ~KeV. So the cyclotron does not operate on lighter particle.

*In India, Cyclotron is available for nuclear research purposes at Variable energy Cyclotron Center (**VECC**), Kolkata.*

3. Synchro-Cyclotron (Modified Cyclotron)

At higher speeds,

$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ and so the resonance condition reduces to $\omega = \frac{Bq}{m_0}\sqrt{1-\frac{v^2}{c^2}}$, $2\pi f' = \omega$. Here f' is the frequency of ion revolution.

$$f' = \frac{qB}{2\pi m_0}\sqrt{1-\frac{v^2}{c^2}}$$

This is the resonance condition.

There are two ways to maintain this condition at higher speeds-

i) The frequency of the alternating field can be reduced while keeping the magnetic field constant. This is done in **synchro-cyclotron**.
ii) Magnetic field B can be increased so that B/m remains constant. This is done in **Betatron**.

The machine in which *B* is kept constant while *f* is decreased continuously is called a synchrocyclotron or a modified cyclotron.

For a typical synchrocyclotron, the cyclotron frequency range is 36-18MHz. The energy of proton is about 750MeV at field strength of about 2.3Tesla.

4. Synchrotron

The resonance condition shows that an alternate procedure to extend cyclotron to higher energies is to increase the magnetic field with increasing orbital radius. Machines in which the magnetic field *B* is varied while the frequency *f* may or may not be varied are called synchrotrons. The system at the Fermi national accelerator laboratory at Illinois has a radius of 1.0km and the one at CERN, Geneva 1.1km. These large synchrotrons do not use magnets 1km in radius. Instead, a narrow ring of magnets is used with each magnet placed at the same radius from the centre of the circle. Once the particles are injected, they must move in a circle of constant radius. For this, the particles have to be accelerated initially in a smaller machine and then the magnetic field is slowly increased as they speed up in the bigger synchrotron. The magnets are interrupted by gaps where a high voltage accelerates the particles. The new Tevatron at Fermilab uses superconducting magnets to accelerate protons to 1 TeV=1000GeV. Since the particles are accelerated to 1TeV, the name Tevatron.

4.1. Drawbacks: Synchrotron Radiation

When charged particles are accelerated, they radiate electromagnetic energy, called synchrotron radiations. The effect is more pronounced in circular machines where centripetal acceleration is present, particularly synchrotrons. If the curvature is small, electrons can radiate as much energy as they gain. To avoid synchrotron radiation losses, the radius is increased to the maximum possible extent.

Though, now a days synchrotron radiation is being used for useful purposes such as a light sources in basic and applied research and in X ray lithography to produce minaturised computer chips.

*In India, two Synchrotrons namely Indus-1 and Indus-2 are available for research purposes at Raja Ramanna Centre for Advanced Technology (**RRCAT**), Indore.*

Problem: Show that the energy of a particle having a charge e in a synchrotron is given by E=Brc (eV) in the relativistic limit. B=Magnetic field, r=radius.

Answer: In the relativistic limit, v~c.

$E=mc^2$, so, $m=E/c^2$

Now, $qvB= mv^2/r$, or, $B=mv/qr=\frac{\left(\frac{E}{c^2}\right)c}{er}$

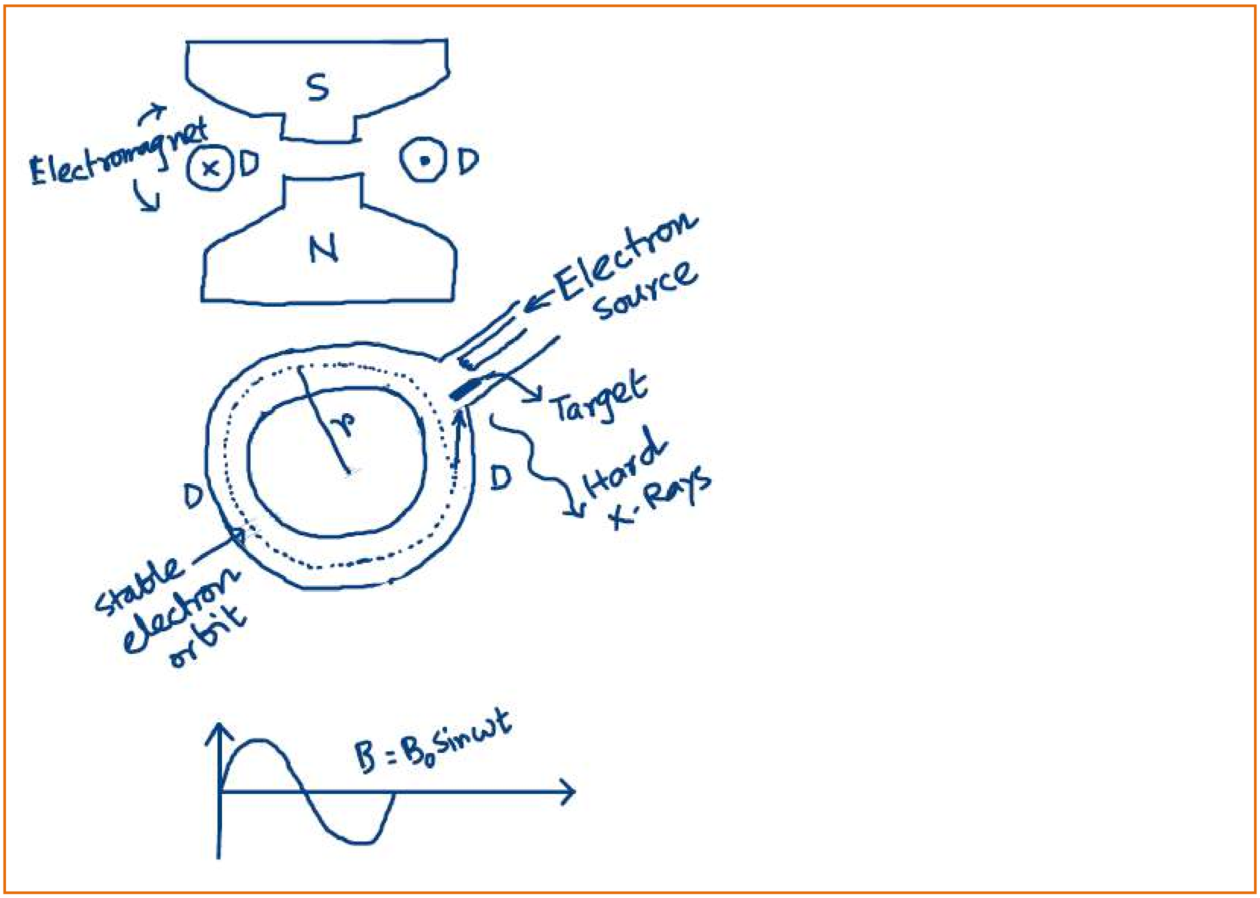
Hence, E=Berc (J)= Brc (MeV)

5. Betatron

This is primarily used to accelerate electrons with mass 0.511MeV and KE 0.05MeV. In a Betatron a beam of electron can be accelerated at energy~300MeV

5.1. Construction:

A large highly evacuated annular tube DD called the donut tube where electrons revolve which are shown entering the page (×) and emerging the page (·). The tube DD is placed between the pole pieces of a large electromagnet. Since the magnet is energized by alternating current, an increasing magnetic field in a given direction is obtained only during one quarter cycle in which the current increases from zero to peak value. Hence the output electron beam is a pulse.



5.2. Betatron Condition:

The radius of circular path of electron is

$$\frac{mv^2}{R} = BeV - (i)$$

$$or, \qquad P = BeR - (ii)$$

Here the particle gains energy by the varying magnetic field $B=B_0\sin\omega t$. This produces a changing magnetic flux $\varphi=\varphi_0\sin\omega t$ linked with the electron orbit. The induced emf per turn is $d\varphi/dt$ and the work done on the electron is $ed\varphi/dt$ which equals the gain in kinetic energy of electron per turn. The energy of the particle is continuously increasing by the electromagnetic induction. It gains approx 500-600eV per turn. The particle has the tendency to increase the radius. The varying magnetic field supplies energy to the electron and simultaneously keeps it in a fixed orbit-provided a condition is satisfied.

If F is the tangential force acting on every electron at every point which accelerates the electron beam,

$$e\frac{d\varphi}{dt} = F2\pi R$$

$$F = \frac{e}{2\pi R}\frac{d\varphi}{dt}$$

Rate of change of linear momentum=Tangential force. So,

$$\frac{dP}{dt} = \frac{e}{2\pi R}\frac{d\varphi}{dt} \quad --(iii)$$

From (ii) and (iii),

$$\frac{d}{dt}(BeR) = \frac{e}{2\pi R}\frac{d\varphi}{dt}$$

$$eR\frac{dB}{dt} = \frac{e}{2\pi R}\frac{d\varphi}{dt}$$

$$\frac{d\varphi}{dt} = 2\pi R^2\frac{dB}{dt} --(iv)$$

This is **Betatron Condition**.

If this condition is satisfied, the particle (electron) will continuously increase its speed while moving in a fixed orbit. Integrating,

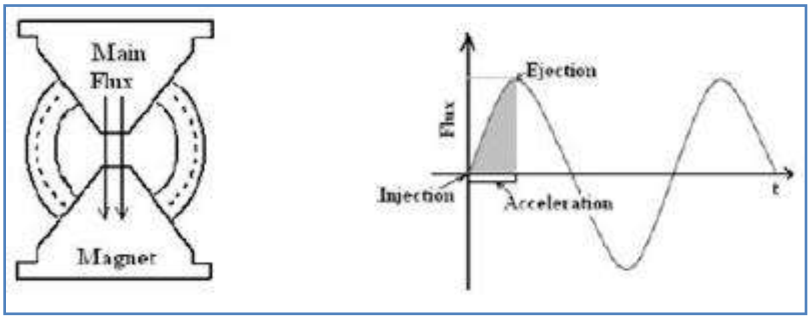
$$\varphi_f - \varphi_i = 2\pi R^2(B_f - B_i)$$

i is the initial value, *f* is the final value. Let at *t=0*, *B=0*, *φ=0*. The change of magnetic flux

$$\varphi_0 = 2\pi R^2 B_0$$

The average kinetic energy gain of electron per turn is

$$\overline{\Delta T} = \langle e\frac{d\varphi}{dt}\rangle = \frac{1}{t}\int_0^t e\frac{d\varphi}{dt}dt$$



As can be seen from the above image, time of acceleration $t = \frac{T}{4} = \frac{\pi}{4} = \frac{\pi}{2\omega} \sim \frac{1}{200}s$

$$\overline{\Delta T} = \frac{2\omega}{\pi}e\int_0^{\frac{\pi}{2\omega}}\frac{d\varphi}{dt}dt = \frac{2\omega e}{\pi}\varphi_0 \sin\omega t\,|_0^{\frac{\pi}{2\omega}} = \frac{2\omega e}{\pi}\varphi_0$$

$$= \frac{2\omega e}{\pi}2\pi R^2 B_0$$

$$= 4B_0\omega e R^2 \sim 500 - 600eV$$

The number of revolutions of electrons per second in a Betatron in the acceleration time T/4 (~1/200 s)

$N = \frac{c}{2\pi R} \times \frac{T}{4}$

v=c, speed of light. Assuming electrons moves near to speed of light which is in fact true after some revolutions.

$$= \frac{c}{2\pi R} \times \frac{\pi}{2\omega}$$

$$= \frac{c}{4\omega R}$$

$$\left[N \sim \frac{3 \times 10^8}{4 \times 100\pi \times 1} \sim 2{,}50{,}000\right]$$

Final energy of electron is

T=$\overline{\Delta T} N$

$=4B_0 \omega e R^2 \frac{C}{4\omega R}$

$=B_0 ecR$ Joules

5.3. Limitation of Betatron:

The maximum kinetic energy of electron is limited by the radiation loss of the electron beam per turn. The radiation loss per second is proportional to 4^{th} power of energy. The maximum energy is attained when the gain of energy per turn is equal to loss of energy per turn by electromagnetic radiations.