CAPITAL ASSET PRICING MODEL (CAPM)

In the following discussion on portfolio theory, we use **σ as a measure of risk**, which is really the standard deviation of returns. Another useful measure of risk is the **β of an investment**. Like σ, β is also a statistical measure of risk. We infer it from the observations of the past performance of a stock. For example, we may want to find the risk of buying and holding the stock of a particular firm, and we are interested in finding the β of that firm. We can start by looking at the historical value of three variables:

1. **The returns of the stock, Rj.** We define the return on a stock by the relation

$$R_j = (P_1 - P_0 + D_1)/ P_0$$

where P_1 is the price at the end of the year, P_0 is the price at the beginning of the year and D_1 is the dividend paid, if any, at the end of the year.

2. **The returns of the market, R_m.** A market index provides an overall measure of the performance of the market. A well-known market index (internationally) is the Dow Jones Industrial Average. For India, we have our Sensex. For a broader market index, we may have to look at S&P100, or S&P500 index (internationally) and NSE-500 on the domestic front.

Therefore, we define the overall return on the market as:

$Rm = [(M_1 - M_0)/ P_0] + d_1$ where d_1 is the dividend yield calculated at the end of the year. Note that the D_1 and d_1 are different: while D_1 is total dividend, d_1 is the percentage of dividend, while M_0 is the beginning value and M_1 the ending value of the market index.

3. **The riskless rate of interest (r).** The securities issued by the government, such as the Treasury bills, bonds, and notes, are, by definition, riskless. They are the safest investments available, backed by the full faith and taxing power of the government. The data on riskless return is thus freely available.

Then we define two variables x and y as:

$y = Rj - r$
$x = Rm - r$

By subtracting the riskless rate of interest, we are able to see the return due to the risk inherent in the given stock, and the return from the risk in the market. Thus, we are comparing the returns **exclusively due to the risk in the investments.**

A regression line drawn between the various observed values of x and y will show a certain (linear) relationship between x and y. The slope of the line will give the rate of change of y with respect to x. In other words, the slope will signify how much the return on the stock will change corresponding to a given change in the return on the market. In this diagram let us say that the slope of the line is β. While the intercept of the line is not very important, the quantity β represents an important concept.

This responsiveness of the stock return to the changing market conditions is called the "beta" of the stock. Stocks with low betas will show very little movement due to the fluctuations in the stock market. High beta stocks will tend to be jumpy showing a large variation in response to small changes in the market.

High β stocks, due to their large volatility, will be more unpredictable, and therefore, more risky. Low beta stocks show relatively small volatility, and they are more predictable and safe.

Beta is a statistical quantity, and it is a measure of the systematic risk, or the market related risk of a stock. These results can also be expressed as a statistical formula,

$$\beta j = \mathrm{cov}(R_j,R_m)/\mathrm{var}(R_m) = r_{jm}\,\sigma_m\sigma_j/\sigma_m^2 = r_{jm}\,\sigma_j/\sigma_m$$

where **cov** (R_j,R_m) is the covariance between market return and return of the individual stock and the **σ** refers to the standard deviation on market return.

One can apply the concept of beta to a portfolio. The beta of a portfolio is simply the weighted average of the betas of the securities in the portfolio,

Beta of a portfolio, $\beta_p = w_1\beta_1 + w_2\beta_2 + w_3\beta_3 + ... = \sum w_i\beta_i$, where the w_i are weights.

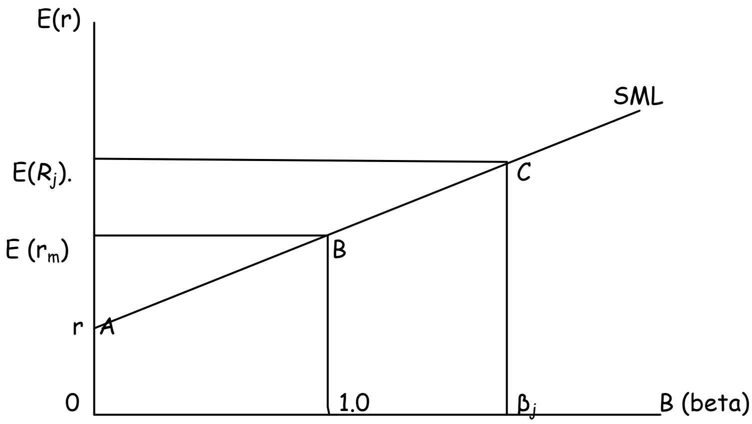
The advantage of using β as a measure of risk is that it can combine linearly for different securities in a portfolio, but the disadvantage is that it can measure only the market related risk of a security. On the other hand, σ can measure the risk independent of the market conditions, but its disadvantage is that it is non-linear in character and difficult to apply in practice. Both β and σ are incomplete measures of risk; they change with time, and are difficult to measure accurately.

By definition, the beta of a riskless investment is zero. Further, the beta of the market is 1.

A security that has a high beta should show a large rise in price when there is an upward movement in the market, and has a large drop in price in case of a downward movement. These large price fluctuations can cause a considerable amount of uncertainty about the return of this security, and greater risk associated with it. Therefore, a high beta security is also a high-risk security. A low beta security is therefore a defensive security and a high beta of a stock represents a more aggressive investment policy.

Beta is a measure of the market risk, or **the systematic risk, of a security**. As a security with a large beta will have large swings in its price in relation to the changes in the market index, it leads to a higher standard deviation in the returns of the security, which will indicate a greater uncertainty about the future performance of the security.

Let us now draw a diagram with the β of various securities along the X-axis and their expected return along the Y-axis. We have already noticed that β is a linear measure of risk. If we *assume* that a linear relationship exists between the risk and return, then only two points are sufficient to draw a straight line in this diagram. The line, representing the relationship between risk and expected return, is called the ***security market line***. Under equilibrium conditions, all other securities will also lie along this line. Higher β securities will have a correspondingly higher expected return. Figure 1 shows this graphically.



By definition, the beta of the market portfolio is equal to 1. Thus, the return corresponding to beta equal to 1 is the market return E (r_m).

The security market line represents the risk-return characteristics of various securities, assuming that there is linear relationship between risk and return. Point *A* represents a riskless security with beta equal to zero and return *r*. Point B shows a market-indexed security which could be a very large mutual fund portfolio. Point *C* shows an individual security whose beta (β_j) is higher than 1 and whose expected return is $E(R_j)$. Since *A*, B, *C* all lie along the same straight line, we have

Slope of segment *AC* = slope of segment *AB*

This gives, $(E(Rj) - r) / \beta_j = (E(Rm) - r) / 1$

Or, $\boxed{E(R_j) = r + \beta_j [E(Rm) - r]}$ ...**(1)**

Equation **(1)** gives the expected return of a security *j* in terms of its risk, expected return on the market, and the riskless rate. It is a forward-looking model, and thus gives the expected values of the returns. This equation represents what is known as the CAPM, and was developed by William Sharpe, Jan Mossin and John Lintner.

A sum

XYZ stock will pay a dividend of Rs.1.32 next year. Its current price is Rs. 24.6 per share. The beta for the stock is 1.35 and the expected return on the market is 13.5%. If the riskless rate is 8.2%, what is the expected growth rate of XYZ?

Using the capital asset pricing model (CAPM),

$\boxed{E(R_j) = r + \beta_j [E(Rm) - r]}$ ...(1), we first find the expected rate of return as

$\boxed{E(R_j)} = 0.082 + 1.35 [0.135 - 0.082] = 0.15355 = R$

The expected rate of return $E(R_j)$, for a security is also its required rate of return R by the investors.

Using the growth model for a stock, $P_0 = D_1 / (R - g)$

we get, $R - g = D_1/P_0$, or $g = (R - D_1)/P_0$,

which gives g approximately equal to 10%