Online Course Materials:

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Topic / Title : Some important Problems and solutions on Relation.

S.K. Mapa

2(d) $S = \mathbb{Z} \times \mathbb{Z}$ and ρ is defined on $\mathbb{Z} \times \mathbb{Z}$ by

$(a,b)\,\rho\,(c,d)$ iff $ad = bc$, $(a,b), (c,d) \in \mathbb{Z}\times\mathbb{Z}$

Then find the nature of ρ.

Solution: Let $(a,b) \in \mathbb{Z}\times\mathbb{Z}$ | $(a,b)\,\rho\,(a,b)$

Then $a \cdot b = b \cdot a$ holds, $\forall$ $(a,b) \in \mathbb{Z}\times\mathbb{Z}$

$\Rightarrow (a,b)\,\rho\,(a,b)$ holds, $\forall (a,b) \in \mathbb{Z}\times\mathbb{Z}$
(by defn of ρ)

$\Rightarrow \rho$ is reflexive.

Let $(a,b), (c,d) \in \mathbb{Z}\times\mathbb{Z}$ and $(a,b)\,\rho\,(c,d)$.

Then $(a,b)\,\rho\,(c,d) \Rightarrow a \cdot d = b \cdot c$

$\Rightarrow c \cdot b = d \cdot a$

$\Rightarrow (c,d)\,\rho\,(a,b)$

$\therefore \rho$ is symmetric.

Let $(a,b), (c,d), (e,f) \in \mathbb{Z} \times \mathbb{Z}$. and

$$(a,b)\, \rho\, (c,d) \quad \& \quad (c,d)\, \rho\, (e,f).$$

then $(a,b)\, \rho\, (c,d)$ & $(c,d)\, \rho\, (e,f) \Rightarrow ad = bc$
& $cf = de$

$$\Rightarrow adcf = bcde$$

$$\not\Rightarrow af = be \quad (\text{when } dc = 0$$

ie $(a,b)\, \bar{\rho}\, (e,f)$

For a counter example,

$$(1,2)\, \rho\, (0,0) \quad \text{and} \quad (0,0)\, \rho\, (2,1)$$

as $1.0 = 2.0$ & $0.1 = 0.2$

But $(1,2)\, \bar{\rho}\, (2,1)$ as $1.1 \neq 2.2$

Thus ρ is not transitive relation. ρ is only reflexive and symmetric relation.

3. Examine if the relation ρ on the set $\mathbb{Z}$ is an equivalence relation

(a) $\rho = \{ (a, b) \mid 3a + 4b \text{ is divisible by } 7 \}$

(b) $\rho = \{ (a, b) \mid |a - b| \leq 3 \}$

Solution (a) Let $a \in \mathbb{Z}$.

Then $3a + 4a = 7a$ is divisible by 7

$, \forall a \in \mathbb{Z}$

$\Rightarrow a \rho_a$ holds, $\forall a \in \mathbb{Z}$

$\Rightarrow \rho$ is reflexive.

Let $a, b \in \mathbb{Z}$ and $a \rho_b$

Then $a \rho_b \Rightarrow 3a + 4b$ is divisible by 7

$\Rightarrow 3a + 4b = 7k$, for some $k \in \mathbb{Z}$.

$\Rightarrow (7a - 4a) + (7b - 3b) = 7k$

$\Rightarrow 3b + 4a = 7(a + b - k)$

$\Rightarrow 3b + 4a = 7k_2$ where $k_2 = a + b - k \in \mathbb{Z}$

$\Rightarrow 3b + 4a$ is divisible by 7

$\Rightarrow b \rho_a$

$\therefore$ ρ is symmetric.

Let $a, b, c \in \mathbb{Z}$ and $a \rho_b$ & $b \rho_c$

Then $a \rho_b$ & $b \rho_c$ $\Rightarrow$ $3a + 4b = 7K_1$ and
$3b + 4c = 7K_2$, for
some $K_1, K_2 \in \mathbb{Z}$

$\Rightarrow$ $3a + 4c + 7b = 7K_1 + 7K_2$

$\Rightarrow$ $3a + 4c = 7(K_1 + K_2 - b)$

$\Rightarrow$ $3a + 4c = 7K_3$ where
$K_3 = K_1 + K_2 - b \in \mathbb{Z}$

$\Rightarrow$ $a \rho_c$.

$\therefore$ ρ is transitive. Thus ρ is an equivalence relation as ρ is reflexive, symmetric and transitive.

(b) Let $a \in \mathbb{Z}$. Then $|a - a| = 0 \leq 3$, $\forall a \in \mathbb{Z}$

$\Rightarrow$ $a \rho_a$ holds, $\forall a \in \mathbb{Z}$

$\Rightarrow$ ρ is reflexive

Let $a, b \in \mathbb{Z}$ and $a \rho_b$. Then $a \rho_b \Rightarrow |a - b| \leq 3$

$\Rightarrow |b - a| \leq 3$

$\Rightarrow b \rho_a$

$\therefore$ ρ is symmetric.

Let $a, b, c \in \mathbb{Z}$ and $a \rho_b$ & $b \rho_c$.

Then $a \rho_b$ & $b \rho_c \Rightarrow |a-b| \leq 3$ & $|b-c| \leq 3$

This may not imply

$|a-c| \leq 3$.

For example, $0 \rho_{-3}$ and $-3 \rho_{-5}$

as $|0-(-3)| = 3 \leq 3$ and

$|-3-(-5)| = |-3+5| = 2 \leq 3$

But $0 \bar{\rho}_{-5}$ as $|0-(-5)| = 5 \nleq 3$.

$\therefore$ ρ is not transitive and hence ρ is not equivalence relation.

<u>Th.</u> Let ρ be an equivalence relation on S

Then $a \rho_b$ iff $cl(a) = cl(b)$.

<u>Proof:</u> Let $a \rho_b$. We shall show that $cl(a) = cl(b)$

Let $x \in cl(a)$. Then $a \rho_x$

$\Rightarrow x \rho_a$ ($\because \rho$ is symmetric)

Since $a \rho_b$, so by transitive property of ρ we get $x \rho_b \Rightarrow b \rho_x \Rightarrow x \in cl(b)$

$\therefore \; cl(a) \subseteq cl(b)$ — (I)

Again let $y \in cl(b)$. Then $b \rho y$

$\Rightarrow y \rho b$ (since ρ is symm).

Since $a \rho b$, so $b \rho a$ (as ρ is symmetric).

Then $y \rho b$ & $b \rho a \Rightarrow y \rho a$

$\Rightarrow a \rho y$

$\Rightarrow y \in cl(a)$

$\therefore \; cl(b) \subseteq cl(a)$ — (II)

From (I) & (II) we get

$cl(a) = cl(b)$

Conversely, let $cl(a) = cl(b)$. We shall show that $a \rho b$

Since $a \in cl(a) = cl(b)$

$\Rightarrow a \in cl(b)$

$\Rightarrow b \rho a$

$\Rightarrow a \rho b$ (since ρ is symmetric)

Hence, the theorem is completed.

11. Let ρ be an equivalence relation on S. Then prove that $a \bar{\rho} b$ iff $cl(a) \cap cl(b) = \Phi$, $a, b \in S$.

Proof: Let $a \bar{\rho} b$. We shall show that

$$cl(a) \cap cl(b) = \Phi.$$

If possible, let $cl(a) \cap cl(b) \neq \Phi$.

Then $x \in cl(a) \cap cl(b)$, $x \in S$

$\Rightarrow x \in cl(a)$ and $x \in cl(b)$

$\Rightarrow a \rho x$ and $b \rho x$

$\Rightarrow a \rho x$ and $x \rho b$ (since ρ is sym)

$\Rightarrow a \rho b$ ($\because \rho$ is transitive

which contradicts the fact that $a \bar{\rho} b$

Thus $cl(a) \cap cl(b) = \Phi$.

Conversely, let $cl(a) \cap cl(b) = \Phi$. We shall show that $a \bar{\rho} b$.

If possible, let $a \rho b$. Then $cl(a) = cl(b)$

Since $a \in cl(a) = cl(b)$, so $a \in cl(b)$

$\Rightarrow a \in cl(a) \cap cl(b)$

$\Rightarrow cl(a) \cap cl(b) \neq \Phi$, which contradicts the fact $cl(a) \cap cl(b) = \Phi$.

Thus $a \, \bar{\rho} \, b$.

Hence the theorem is completed.

Th. Let S be a non empty set and ρ be an equivalence relation on S. Then ρ yields a partition of S.

Proof: Since ρ is an equivalance relation on S. So we calculate $cl(a)$, where $a \in S$

$$\text{Let } A = \{ cl(a) \mid a \in S \}$$

Since we know that any two classes are either identical or disjoint. So, we collect all disjoint classes from the set A.

Let $B = \{ cl(a_i) \mid i \in I \}$ (where I is any index set) be the collection of all distincts classes.

Then $\bigcup\limits_{i \in I} cl(a_i) = \bigcup\limits_{a \in S} cl(a)$ ---- (1)

Since $\bigcup\limits_{a \in S} cl(a) = S$, so from (1) we get

$$\bigcup_{i \in I} cl(a_i) = \bigcup_{a \in S} cl(a) = S$$

$$\Rightarrow \bigcup_{i \in I} cl(a_i) = S \quad \cdots (ii)$$

Again as $cl(a_i)$ $(i \in I)$ are all distinct

so $cl(a_i) \cap cl(a_j) = \phi$, $(i \neq j)$ - (iii)

In view of (ii) & (iii), we say that $B = \{ cl(a_i) \mid i \in I \}$ form a partition of S.

Th: Let S be a non empty set and $\mathcal{A} = \{ A_i \mid i \in I \}$, where I is any index set, be a partition of S. Then show that there exist an equivalence relation on S

Proof: We define a relation ρ on S as follows

$x, y \in S$, $x \rho y$ iff $x, y \in A_i$, for some $i \in I$

Let $a \in S$. Since $\mathcal{A} = \{ A_i \mid i \in I \}$ form a partition of S, so $\exists$ unique $K \in I$ s.t

$a \in A_K$

$\Rightarrow a, a \in A_K$

$\Rightarrow a \rho a$ holds, $\forall a \in S$. So ρ is reflexive.

Let $a, b \in S$ s.t $a \rho_b$.

then $a \rho_b \Rightarrow a, b \in A_i$, for some $i \in I$

$\Rightarrow b, a \in A_i$, for some $i \in I$

$\Rightarrow b \rho_a$.

$\therefore \rho$ is symmetric.

Let $a, b, c \in S$ s.t $a \rho_b$ & $b \rho_c$

then $a \rho_b \Rightarrow a, b \in A_i$ for some $i \in I$

& $b \rho_c \Rightarrow b, c \in A_j$ for some $j \in I$

Then we find that $b \in A_i$ & $b \in A_j$. Since $\mathcal{A} = \{A_i \mid i \in I\}$ form a partition of S, so it follows that $A_i = A_j$, thus

~~$a, b \in$~~ $a \rho_b$ & $b \rho_c \Rightarrow a, b \in A_i$ and $b, c \in A_i$, for some $i \in I$

$\Rightarrow a, c \in A_i$, for some $i \in I$

$\Rightarrow a \rho_c$.

Thus ρ is transitive and hence ρ is an equivalence relation on S. Then, the theorem is complete.

C-H-2008 Give an example (with justification) of a relation which is symmetric but neither reflexive nor transitive.

Solution: Let $S = \{1, 2, 3\}$.

Then $S \times S = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

Let $\rho = \{ , (2,2)\ (1,3)\ (3,1)\}$

Then ρ is a subset of $S \times S$. So, ρ is relation on S.

(i) Since $1 \in S$ and $1 \bar{\rho} 1$, so ρ is not reflexive.

(ii) Since $a \rho b \Rightarrow b \rho a$ where $a, b \in S$, so ρ is symmetric.

(iii) Since $1 \rho 3$ & $3 \rho 1$, but $1 \bar{\rho} 1$, so ρ is not transitive.

S.K. Mapa

(2). Find the nature of ρ where ρ is defined on $\mathbb{Z}$ by "$a \rho b$ iff $a - b < 3$" where $a, b \in \mathbb{Z}$.

Solution: (i) Let $a \in \mathbb{Z}$

Then $a - a = 0 < 3 \; \forall a \in \mathbb{Z}$

so by def^n of ρ, $a \rho a$ holds, $\forall a \in \mathbb{Z}$.

Hence ρ is reflexive.

(ii) Let $a, b \in \mathbb{Z}$ s.t $a \rho b$

Then $a \rho b \Rightarrow a - b < 3$

$\not\Rightarrow b - a < 3$ (always)

ie $a \rho b \not\Rightarrow b \rho a$, (always) for example

$0 \rho 7$ as $0 - 7 = -7 < 3$ but

$7 \bar{\rho} 0$ as $7 - 0 = 7 \nless 3$

$\therefore$ ρ is not symmetric.

(iii) Let $a, b, c \in \mathbb{Z}$ s.t $a \rho_b$ & $b \rho_c$

Then $a \rho_b$ and $b \rho_c \Rightarrow a - b < 3$ and $b - c < 3$

$\not\Rightarrow a - c < 3$ (always)

i.e $a \rho_b$ and $b \rho_c \not\Rightarrow a \rho_c$ (always),

for example $6 \rho_4$ and $4 \rho_2$ as $6 - 4 = 2 < 3$

and $4 - 2 = 2 < 3$, but $6 \bar{\rho}_2$ as

$6 - 2 = 4 \nless 3$.

$\therefore$ ρ is not Transitive

Then ρ is only reflexive relation on $\mathbb{Z}$.

2(a) Let S be the set of all lines on a plane and ρ is defined on S by " $l \rho_m$ iff l is perpendicular to m" where $l, m \in S$. Find the nature of ρ

Solution: (i) Let $l \in S$. Then l is not perpendicular to itself, so $l \bar{\rho}_l$, $\forall l \in S$

ie ρ is not reflexive on S

(ii) Let $l, m \in S$ and $l \rho m$

Then $l \rho m \Rightarrow l$ is perpendicular to m

$\Rightarrow m$ is perpendicular to l

$\Rightarrow m \rho l$

So, ρ is symmetric on S.

(iii) Let $l, m, n \in S$ and $l \rho m$ & $m \rho n$

Then $l \rho m$ & $m \rho n \Rightarrow l$ is perpendicular to m

and m is perpendicular to n

$\Rightarrow l$ is parallel to n

$\Rightarrow l$ is not perpendicular to n

$\Rightarrow l \bar{\rho} n$

So ρ is not transitive.

(Counter example : Let l : $x = 0$, m : $y = 0$

n : $x = 1$ $\in S$

Since l is perpendicular to m, so $l \rho m$.

again as m is perpendicular to n, so $m \rho n$,

but as l & n are parallel, not perpendicular

so $l \bar{\rho} n$)

1 (iii) a relation ρ on $\mathbb{Z}$ defined by " $a \rho b$ iff a^2+b^2 is a multiple of 2" where $a, b \in \mathbb{Z}$. Then find the nature of ρ.

Solution: (i) Let $a \in \mathbb{Z}$. Then a^2+a^2
$= 2a^2$, a multiple of 2, $\forall a \in \mathbb{Z}$

So $a \rho a$ holds, $\forall a \in \mathbb{Z}$

$\therefore$ ρ is reflexive,

(ii) Let $a, b \in \mathbb{Z}$ and $a \rho b$

Then $a \rho b \Rightarrow a^2+b^2 = 2K$, for some $K \in \mathbb{Z}$

$\Rightarrow b^2+a^2 = 2K$, where $K \in \mathbb{Z}$

$\Rightarrow b \rho a.$

So, ρ is symmetric.

(iii) Let $a, b, c \in \mathbb{Z}$ and $a \rho b$ and $b \rho c$

Now, $a \rho b$ and $b \rho c \Rightarrow a^2 + b^2 = 2k_1$ & $b^2 + c^2 = 2k_2$

where $k_1, k_2 \in \mathbb{Z}$

$\Rightarrow a^2 + 2b^2 + c^2 = 2k_1 + 2k_2$

$\Rightarrow a^2 + c^2 = 2(k_1 + k_2 - b^2)$

$\Rightarrow a^2 + c^2$ is a multiple of 2 as $k_1 + k_2 - b^2 \in \mathbb{Z}$

$\Rightarrow a \rho c$

So, ρ is transitive. Since ρ is reflexive, symmetric and transitive, so ρ is an equivalence relation on $\mathbb{Z}$.

7 (i) Let S be a finite set containing two elements. How many different binary relation can be defined on S? How many of these are reflexive?

(ii) Show that the number of different reflexive relation on a set of n elements is $2^{n^2 - n}$.

Solution: (i) Since S contains two elements, so $S \times S$ contains $2 \times 2 = 4$ elements. Since any subset of $S \times S$ is a relation on S, so there are $2^4 = 16$ binary relation on S.

2nd part: Since any reflexive relation ρ on S contains the elements (a, a), (b, b) where $S = \{a, b\}$, so there are only 2 elements namely (a, b), (b, a) in $S \times S$, which may or may not be in ρ. So, total no of reflexive relation is ${}^2C_0 + {}^2C_1 + {}^2C_2$

$= 4$.

(ii) Since S contain n elements, So let $S = \{a_1, a_2 \ldots a_n\}$

then $S \times S$ contains $n \times n = n^2$ elements.

Since any reflexive relation ρ on S must contains $(a_1, a_1), (a_2, a_2), (a_3, a_3) \ldots (a_n, a_n)$

So there ~~any~~ are $(n^2 - n)$ elements which may or may not be in ρ.

So, total no of reflexive relation is

$$^{(n^2-n)}C_0 + {}^{(n^2-n)}C_1 + {}^{(n^2-n)}C_2 + \cdots + {}^{(n^2-n)}C_{(n^2-n)}$$

$$= (1+1)^{n^2-n} = 2^{n^2-n} \text{ (proved).}$$

S.K. Mapa

4 A relation β is defined on $\mathbb{Z}$ by $x \beta y$ iff $x^2 - y^2$ is divisible by 5, ~~x~~, $x, y \in \mathbb{Z}$. Prove that β is an equivalence relation on $\mathbb{Z}$. Show that there three distincts equivalence classes.

Solution: (i) Let $x \in \mathbb{Z}$. Then $x^2 - x^2 = 0$ is divisible by 5, $\forall x \in \mathbb{Z}$

$\Rightarrow x \beta x$ ~~holds~~, $\forall x \in \mathbb{Z}$

So β is reflexive

(ii) Let $x, y \in \mathbb{Z}$ and $x \beta y$

Then $x \beta y \Rightarrow x^2 - y^2 = 5K$, for some $K \in \mathbb{Z}$

$\Rightarrow y^2 - x^2 = 5(-K)$

$\Rightarrow y^2 - x^2$ is divisible by 5 as $-K \in \mathbb{Z}$

$\Rightarrow y \beta x$.

So β is symmetric

(iii) Let $x, y, z \in \mathbb{Z}$ and $x \beta y$ & $y \beta z$

Then $x \beta y$ & $y \beta z \Rightarrow x^2 - y^2 = 5k_1$ & $y^2 - z^2 = 5k_2$

where $k_1, k_2 \in \mathbb{Z}$

$$\Rightarrow x^2 - z^2 = 5(k_1 + k_2)$$

$$\Rightarrow x^2 - z^2 \text{ is divisible by } 5$$

as $k_1 + k_2 \in \mathbb{Z}$

$$\Rightarrow x \beta z$$

So β is transitive. As β is reflexive, symmetric and transitive, so β is an equivalence relation on $\mathbb{Z}$.

<u>2nd part</u>: $cl(0) = \{x \in \mathbb{Z} \mid x \beta 0\}$

$= \{x \in \mathbb{Z} \mid x^2 \text{ is divisible by } 5\}$

$= \{0, \pm 5, \pm 10, \pm 15 \ldots\}$

$cl(1) = \{x \in \mathbb{Z} \mid x \beta 1\}$

$= \{x \in \mathbb{Z} \mid x^2 - 1 \text{ is divisible by } 5\}$

$= \{ \pm 1, \pm 4, \pm 6, \pm 9, \pm 11, \cdots \}$

$$C(2) = \{ x \in \mathbb{Z} \mid x \beta_2 \}$$

$$= \{ x \in \mathbb{Z} \mid x^2 - 4 \text{ is divisible by } 5 \}$$

$$= \{ \pm 2, \pm 3, \pm 7, \pm 8, \pm 12, \pm 13 \cdots \}$$

$$C(3) = \{ x \in \mathbb{Z} \mid x \beta_3 \}$$

$$= \{ x \in \mathbb{Z} \mid x^2 - 9 \text{ is divisible by } 5 \}$$

$$= \{ \cancel{x \in \mathbb{Z}} \ \pm 2, \pm 3, \pm 7, \pm 8, \cdots \}$$

$$= C(2)$$

$$C(4) = \{ x \in \mathbb{Z} \mid x \beta_4 \}$$

$$= \{ x \in \mathbb{Z} \mid x^2 - 16 \text{ is divisible by } 5 \}$$

$$= \{ \pm 1, \pm 4, \pm 6, \pm 9 \cdots \}$$

Then we see that $cl(0)$, $cl(1)$ & $cl(2)$ are disjoint. Also we see that

$$cl(0) \cup cl(1) \cup cl(2) = \mathbb{Z},$$ so the set

of complete distinct classes ~~are cl(0)~~ is $\{cl(0), cl(1), cl(2)\}$. Hence there are three distinct classes.

14 (a) Find the domain and range of the binary relation
$A = \{ (a,b) \mid b \geq a^2;\ a, b \in \mathbb{R} \}$

Solution: Since $b \geq a^2$, so for any real no a, $b \geq 0$.
So, domain of $A = \{ a : a \in \mathbb{R} \} = \mathbb{R}$ and co-domain of $A = \{ b \in \mathbb{R} : b \geq 0 \} = \mathbb{R}^+ \cup \{0\}$, where $\mathbb{R}^+$ is the set of all positive real nos.

(b) Prove that intersection of two equivalence relation on a non empty set A is an equivalence relation. What happens if we consider union instead of intersection?

Solution: Let ρ_1 and ρ_2 be two equivalence relation on a non empty set A. We shall show that $\rho_1 \cap \rho_2$ is equivalence relation on A.

Since ρ_1 & ρ_2 are both equivalence relation on A, so $(a,a) \in \rho_1$ and $(a,a) \in \rho_2$, $\forall a \in A$
$\Rightarrow (a,a) \in \rho_1 \cap \rho_2$, $\forall a \in A$.
$\Rightarrow \rho_1 \cap \rho_2$ is reflexive.

Let $a, b \in A$ and $(a,b) \in \rho_1 \cap \rho_2$. Then $(a,b) \in \rho_1$ and ρ_2
Then $(b,a) \in \rho_1$ and ρ_2 ($\because \rho_1$ & ρ_2 both are ~~not~~ symmetric)
$\Rightarrow (b,a) \in \rho_1 \cap \rho_2$
$\Rightarrow \rho_1 \cap \rho_2$ is symmetric.

Let $a, b, c \in A$ and $(a,b) \in \rho_1 \cap \rho_2$ and $(b,c) \in \rho_1 \cap \rho_2$.
Then $(a,b), (b,c) \in \rho_1$ and ρ_2
$\Rightarrow (a,c) \in \rho_1$ and ρ_2 ($\because \rho_1$ & ρ_2 both are transitive)

$\Rightarrow (a,c) \in \rho_1 \cap \rho_2$

$\Rightarrow \rho_1 \cap \rho_2$ is Transitive

Thus $\rho_1 \cap \rho_2$ is an equivalence relation on A as it is reflexive, symmetric and transitive.

2nd part : If we consider union instead of intersection, the result may be false.

For example, we consider $A = \{1,2,3\}$ and $\rho_1 = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$, $\rho_2 = \{(1,1),(2,2),(3,3),(1,3),(3,1)\}$

Then ρ_1 & ρ_2 are two equivalence relation on A, but

$\rho_1 \cup \rho_2 = \{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1)\}$

is not transitive as $(2,1),(1,3) \in \rho_1 \cup \rho_2$ but $(2,3) \notin \rho_1 \cup \rho_2$

Thus $\rho_1 \cup \rho_2$ is not an equivalence relation on A.

2. (a) S is the set of all lines on a plane and ρ is defined on S by "$l \rho m$ iff 'l is perpendicular to m'" for $l, m \in S$.

Solution: (i) let $l \in S$. Then l is not perpendicular to l, so $l \bar{\rho} l$, $\forall l \in S$.

$\therefore \rho$ is not reflexive relation on S.

(ii) Let $l \rho m$, $l, m \in S$.

Then $l \rho m \Rightarrow l$ is perpendicular to m

$\Rightarrow m$ is perpendicular to l

$\Rightarrow m \rho l$

$\therefore \rho$ is symmetric relation on S.

(iii) let $l \rho m$ and $m \rho n$, $l, m, n \in S$.

Then $l \rho m$ and $m \rho n \Rightarrow l$ is perpendicular to m and m is perpendicular to n

$\not\Rightarrow l$ is ~~perpendicular~~ parallel to n always as l, m, n are coplanar

$\not\Rightarrow l \bar{\rho} n$

$\not\Rightarrow l \rho n$

$\therefore$ ρ is not transitive.

Thus ρ is ~~only~~ symmetric on S but neither reflexive nor transitive.